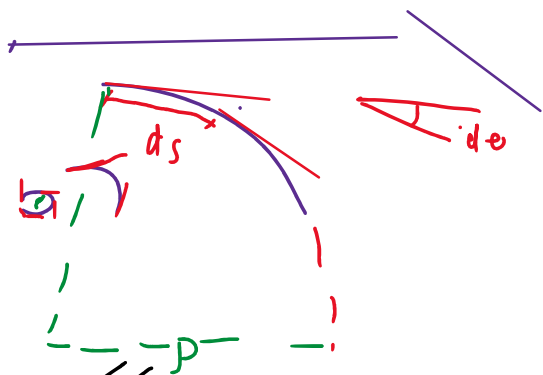


فصل سوم: محاسب تغییر شکل ناشی از جفتش برون‌سای غیر انحراف

انتگرال مضاعف (مقاومت معالج)



- ① کنگر سطح
- ② تغییر زوج

رابطه کنگر-انحناء (moment - Curvature)

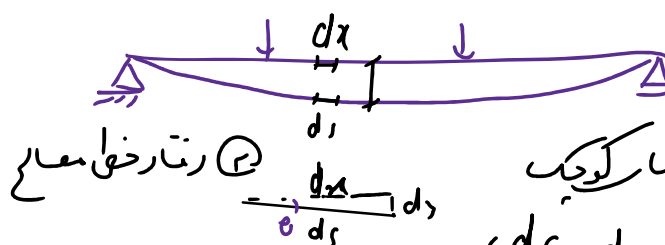
تعریف هندسی

$$\phi = \frac{1}{\rho} = \frac{d\theta}{ds} = \frac{dy'}{dx} = y'' \quad (1)$$

تعریف مساوات
مقاومت

$$\phi = \frac{1}{\rho} = \frac{\epsilon}{r} = \frac{\sigma}{Ey} = \frac{My}{EI} = \frac{M}{EI} \quad (2)$$

$$y'' = \frac{M}{EI}$$



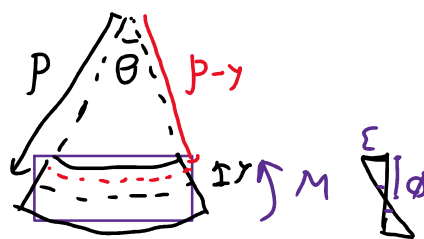
① تغییر شکل کوچک

$$\begin{cases} ds = dx \\ \theta = \tan \theta = y' \end{cases}$$

$$\epsilon = \frac{\Delta L}{L} = \frac{(\rho - y)\theta - \rho\theta}{\rho\theta}$$

$$\epsilon = -\frac{y}{\rho}$$

$$\phi = \frac{1}{\rho} = \frac{\epsilon}{y} \quad \sigma = \frac{My}{I}$$



$$y'' = \frac{M}{EI}$$

$$* \theta = y' = \int \frac{M}{EI} dx$$

$$* y = \int \theta dx$$

$$\frac{dy}{dx} = w \quad \frac{dM}{dx} = v \quad M = \int \sigma y dA = \int \frac{y}{c} \sigma_{max} y dA$$

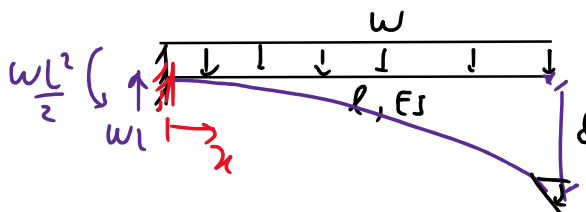
$$v = \int w dx$$

$$M = \int v dx$$

$$M = \frac{\sigma_{max}}{c} \int y^2 dA \quad I$$

$$\theta = \int \frac{M}{EI} dx = \frac{1}{EI} \int M dx$$

$$\theta = \frac{1}{EI} \left(-\frac{wL^2}{2} x + wLx^2 - \frac{wx^3}{6} \right)$$



مثال:

$$\frac{wL^2}{2} \left(\uparrow \right)$$

$$\theta = \frac{1}{EI} \left(-\frac{wL^2}{2}x + \frac{wL}{2}x^2 - \frac{w}{6}x^3 + C_1 \right)$$

$$x=0 \rightarrow \theta=0$$

$$y = \int \theta dx$$

$$y = \frac{1}{EI} \left(-\frac{wL^2}{4}x^2 + \frac{wL}{6}x^3 - \frac{w}{24}x^4 + C_2 \right)$$

$$x=0 \rightarrow y=0 \quad \frac{-6+4-1}{24} = -\frac{3}{24}$$

$$\frac{wL^2}{2} \int \left(\frac{1}{x} \right) dx$$

$$M(x) = -\frac{wL^2}{2} + wLx - \frac{w}{2}x^2$$

$$\theta = -\frac{wL^3}{6EI} \quad \delta = -\frac{wL^4}{8EI}$$

روش کنتر-سطح Moment-area

زود بندر تدریس جارزترین در ۱۸۷۳ میلادی

$$y'' = \frac{M}{EI} \rightarrow \frac{dy'}{dx} = \frac{d\theta}{dx} = \frac{M}{EI}$$

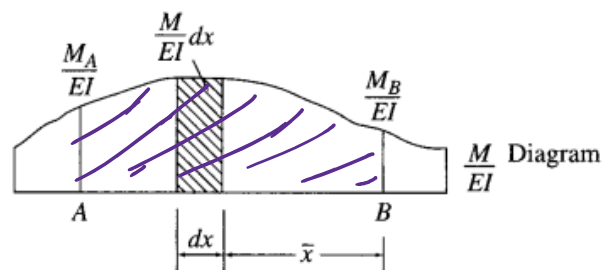
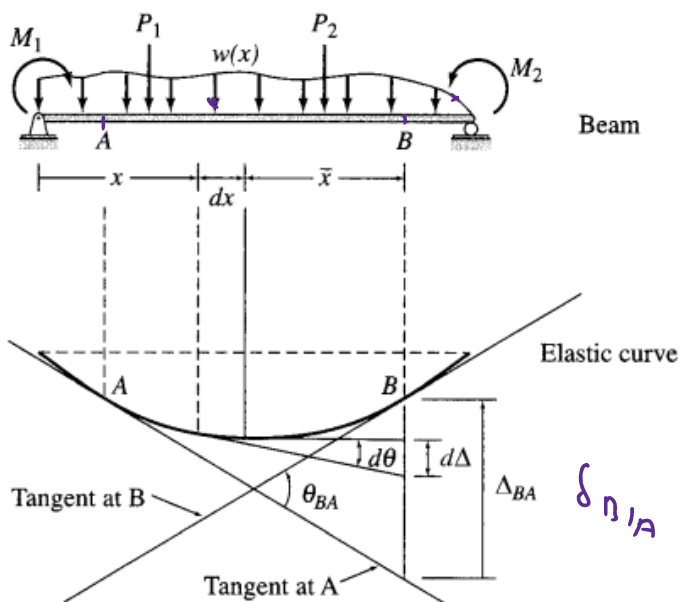
$$d\theta = \frac{M}{EI} dx$$

$$\int_{\theta_A}^{\theta_B} d\theta = \int_{x_A}^{x_B} \frac{M}{EI} dx$$

$$\theta_{B/A} = \int_{x_A}^{x_B} \frac{M}{EI} dx = S \quad \text{تضییع الی}$$

$\theta_{B/A}$: چرخش نقطه B نسبت به نقطه A (بر اساس جهت مثبت مشکلات)

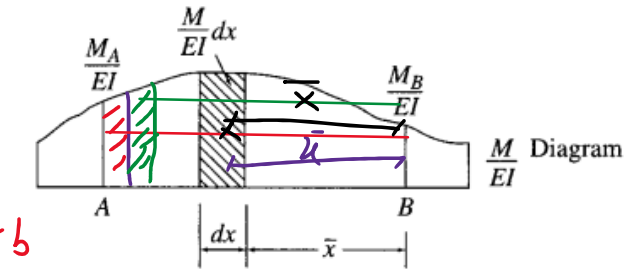
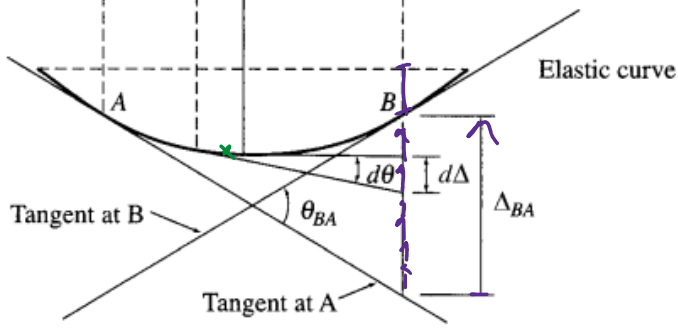
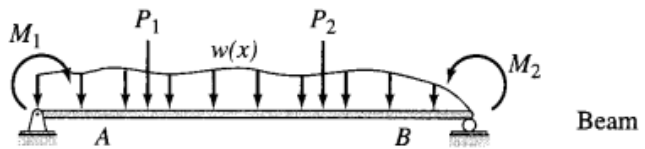
S: مساحت زیر منحنی $\frac{M}{EI}$



$$d\delta = \bar{x} d\theta = \bar{x} \frac{M}{EI} dx$$

$$\delta_{B/A} = \int_{x_A}^{x_B} \bar{x} \frac{M}{EI} dx = \int \bar{x}$$

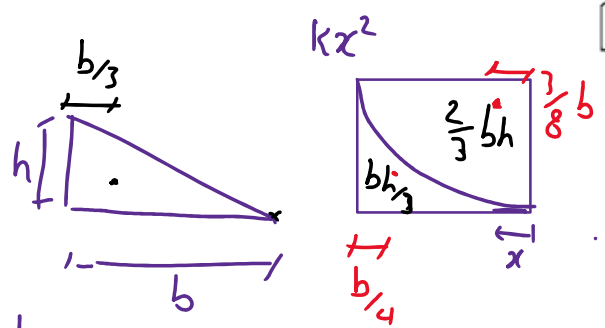
نقطه دوم



$\delta_{B/A}$: انحراف نقطه B از محاس بر A (بر اساس بالا مثبت)

S: مساحت زیر نمودار $\frac{M}{EI}$

\bar{x} : فاصله مرکز سطح $\frac{M}{EI}$ تا B



$$S = b h \frac{1}{2}$$

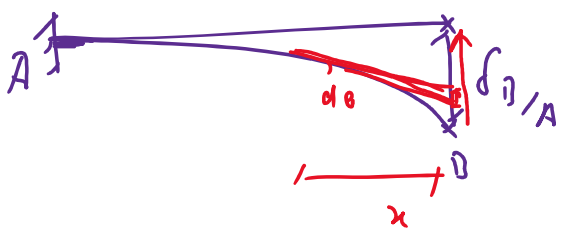
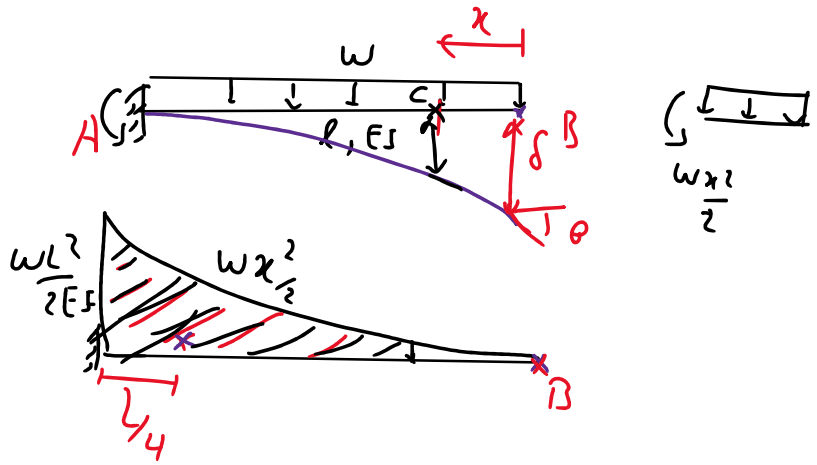
$$\theta_{B/A} = \theta_B - \theta_A = S =$$

$$\frac{1}{3} \left(\frac{wL^2}{2EI} \right) L$$

$$\theta_B = \frac{wL^3}{6EI}$$

$$\delta_{B/A} = \delta_B = \left(\frac{wL^3}{6EI} \right) \left(\frac{3}{4} L \right)$$

$$\delta_B = \frac{wL^4}{8EI}$$



$$\delta_{11} = \frac{wL^4}{8EI}$$

