Deflection Energy3

Thursday, November 23, 2023 11:32

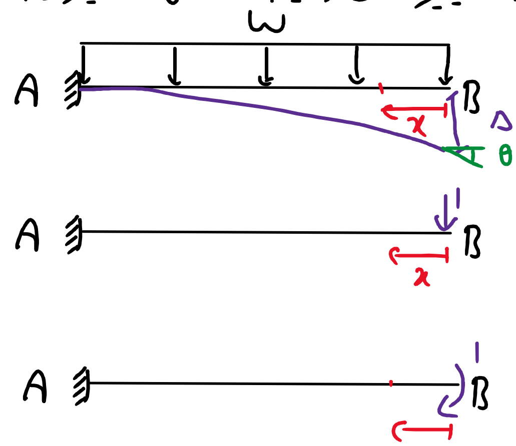
مثال: تغییر مکان و شیب انتهای آزاد تیر را به دست آورید.

$$1 \times \Delta = \int_0^L \frac{mM}{EI}\,dx$$

$$M = -\frac{wx^2}{2}$$

$$m = -x$$

$$m = -1$$

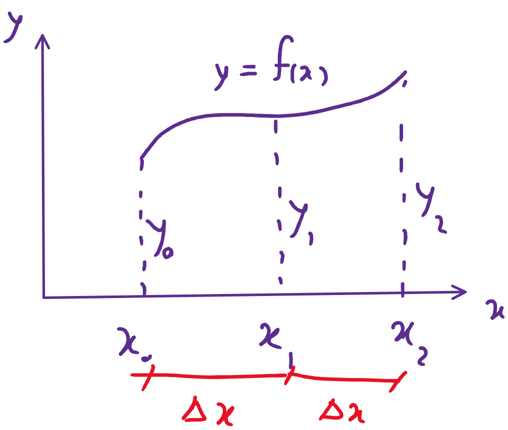


$$1 \times \Delta = \frac{1}{EI}\int_0^L (-x)\left(-\frac{wx^2}{2}\right)dx =$$

$$1 \times \Delta = \frac{w}{2EI}\,\frac{x^4}{4}\Big|_0^L = \boxed{\frac{wL^4}{8EI}}$$

$$1 \times \theta = \int_0^L \frac{mM}{EI}\,dx = \frac{1}{EI}\int_0^L (-1)\left(-\frac{wx^2}{2}\right)dx = \frac{w}{2EI}\int_0^L x^2\,dx = \boxed{\frac{wL^3}{6EI}}$$

محاسبه عددی انتگرال $\int \overset{1}{m}\overset{2}{M}\,dx$



① روش پریزموئیدال

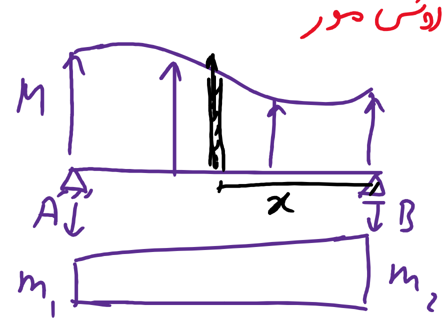
$$\boxed{\int_{x_0}^{x_2} f(x)\,dx = \frac{\Delta x}{3}\left(y_0 + 4y_1 + y_2\right)}$$

سیمسون $\quad \frac{\Delta x}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + y_n\right)$

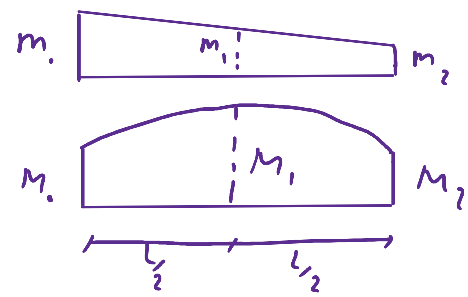
* اگر درجه عبارت زیر انتگرال ۳ یا کمتر باشد، جواب روش پریزموئیدال دقیق است.

② روش مور

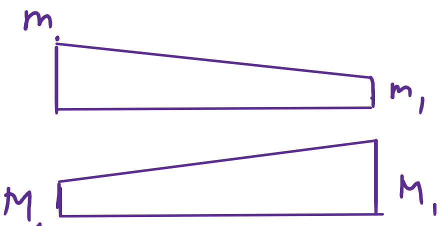
$$\int mM\,dx = m_1 A + m_2 B$$



$$\boxed{\int mM\,dx = \frac{L}{6}\left[m_0M_0 + 4m_1M_1 + m_2M_2\right]}$$

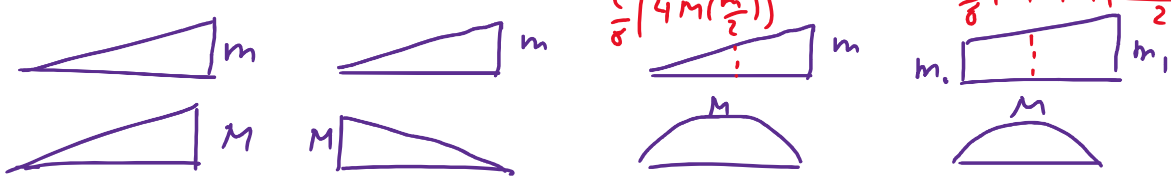


$$\int mM\,dx = \frac{L}{6}\left[m_0M_0 + \cancel{4}\left(\frac{m_0+m_1}{\cancel{2}}\right)\left(\frac{M_0+M_1}{\cancel{2}}\right) + m_1M_1\right]$$



$$\boxed{\int mM\,dx = \frac{L}{6}\left[2m_0M_0 + m_0M_1 + m_1M_0 + 2m_1M_1\right]}$$

$\frac{L}{6}\left(4M\left(\frac{m}{2}\right)\right)$ $\qquad$ $\frac{L}{6}\left(0+4M\left(\frac{m_1+m_2}{2}\right)+0\right)$

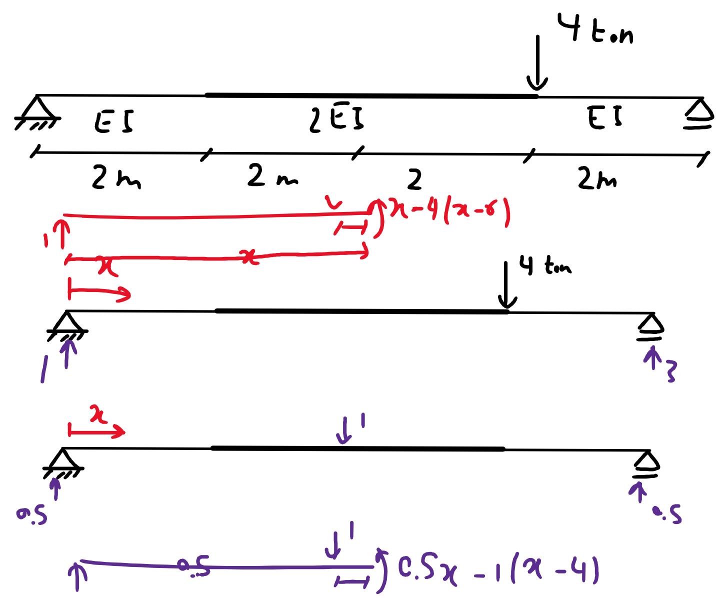


$\frac{L}{3}mM$ $\qquad$ $\frac{L}{6}mM$ $\qquad$ $\frac{L}{3}mM$ $\qquad$ $\frac{L}{3}M(m_1+m_2)$

مثال: تغییر مکان قائم وسط دهانه تیر را بدست آورید ($EI = 6000\, tm^2$)

$$1\times\delta = \int_0^L \frac{mM}{EI}\,dx$$

$$\cancel{t}\times m = \frac{(\cancel{t.m})^2}{\cancel{t.m^2}}\,m$$



$$M = \begin{cases} x & x \le 6 \\ -3x+24 & 6 < x < 8 \end{cases}$$

$$m = \begin{cases} \frac{1}{2}x & x \le 4 \\ -\frac{1}{2}x+4 & 4 \le x \le 8 \end{cases}$$

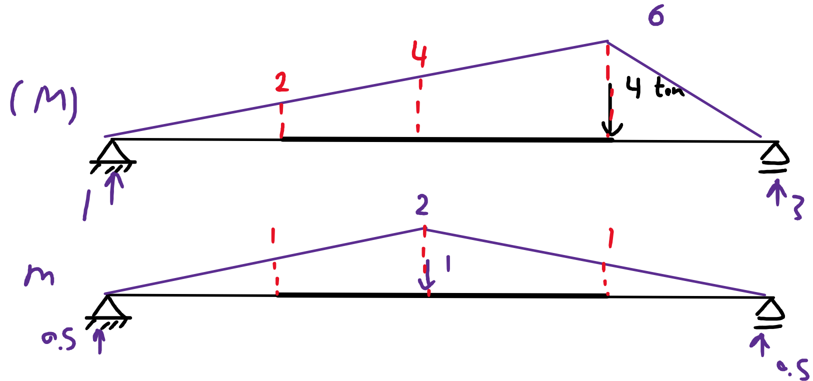
$$1\times\delta = \frac{1}{EI}\int_0^2 \left(\frac{x}{2}\right)(x)\,dx + \frac{1}{2EI}\int_2^4 \left(\frac{x}{2}\right)(x)\,dx + \frac{1}{2EI}\int_4^6 \left(-\frac{x}{2}+4\right)(x)\,dx + \frac{1}{EI}\int_6^8 \left(-\frac{x}{2}+4\right)(-3x+24)\,dx$$

$$1\times\delta = \frac{1}{EI}\left[\frac{x^3}{6}\Big|_0^2 + \frac{1}{2}\,\frac{x^3}{6}\Big|_2^4 + \frac{1}{2}\left(-\frac{x^3}{6}+2x^2\right)\Big|_4^6 + \left(\frac{x^3}{2}-12x^2+96x\right)\Big|_6^8\right]$$

$$1\times\delta = \frac{1}{EI}\left[\frac{x^3}{6}\Big|_0^2 + \frac{1}{2}\,\frac{x^3}{6}\Big|_2^4 + \frac{1}{2}\left(-\frac{x^3}{6}+2x^2\right)\Big|_4^6 + \left(\frac{x^3}{2}-12x^2+96x\right)\Big|_6^8\right]$$

$$1\times\delta = \frac{1}{EI}\left[1.33 + 4.67 + 7.33 + 4\right] = \boxed{\frac{17.33}{EI} = \frac{17.33}{600} = 0.029\,m = 2.9\,Cm}$$

$$\frac{1}{2EI}\left(\frac{2}{6}\right)\left(4\times2+4\times5\times\frac{3}{2}+6\times1\right)$$



$$1\times\delta = \int\frac{mM}{EI}dx = \frac{1}{EI}\left(\frac{2}{6}\right)(2)(1) + \frac{1}{2EI}\left(\frac{2}{6}\right)\left[2(2)(1)+2(4)(2)+(2)(2)+(4)(1)\right]$$

$$+ \underbrace{\frac{1}{2EI}\left(\frac{2}{6}\right)\left[2(4)(2)+2(6)(1)+(4)(1)+(6)(2)\right]} + \frac{1}{EI}\left(\frac{2}{3}\right)(6)(1)$$

$$1\times\delta = \frac{1}{EI}\left[1.33+4.67+7.33+4\right] = \boxed{\frac{17.33}{EI} = 2.9\,Cm}$$