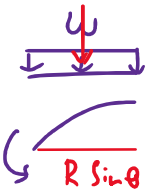
Deflection Energy9

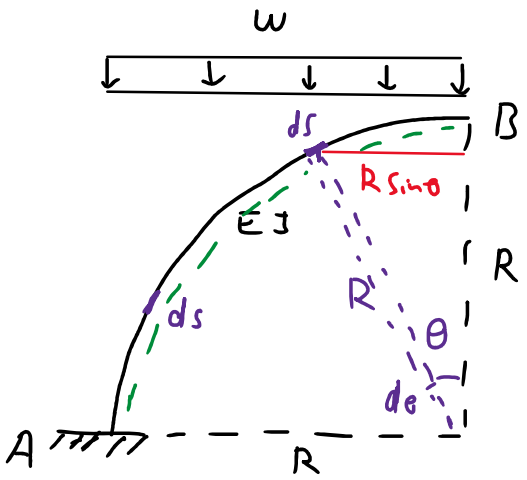
Wednesday, November 29, 2023 16:54

مثال: مطلوب است تغییر مکان قائم و افق نقطه B .

$$1 \times \delta_{By} = \int \frac{mM}{EI} ds$$



$$M = -W\frac{(R\sin\theta)^2}{2}$$

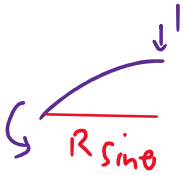


$$1 \times \delta_{By} = \frac{1}{EI}\int_0^{\frac{\pi}{2}} (-R\sin\theta)\left(-\frac{w}{2}R^2\sin^2\theta\right) R\,d\theta$$

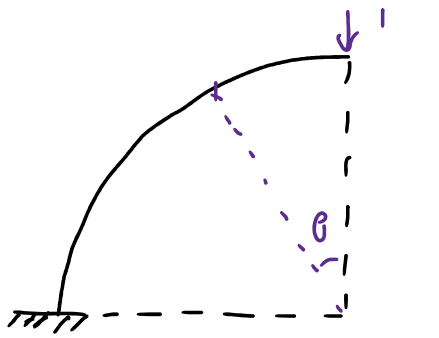
$$\delta_{By} = \frac{wR^4}{2EI}\int_0^{\frac{\pi}{2}} \sin\theta\,(1-\cos^2\theta)\,d\theta$$

$$= \frac{wR^4}{2EI}\left[-\cos\theta + \frac{\cos^3\theta}{3}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{wR^4}{2EI}\left[0 - \left(-1+\frac{1}{3}\right)\right]$$



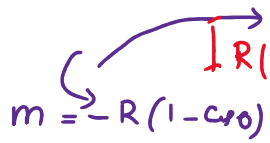
$$m = -R\sin\theta$$



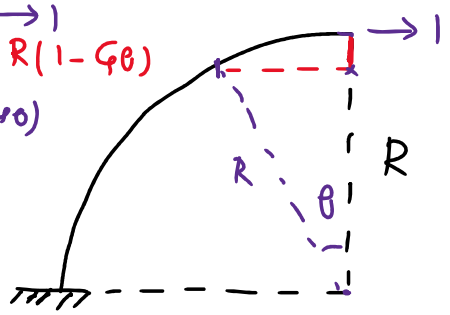
$$\boxed{\delta_{By} = \frac{wR^4}{3EI}}$$

$$1 \times \delta_{Bx} = \int \frac{mM}{EI} ds = \frac{1}{EI}\int_0^{\frac{\pi}{2}} \left(-R(1-\cos\theta)\right)\left(-\frac{w}{2}R^2\sin^2\theta\right) R\,d\theta$$

$$\delta_{Bx} = \frac{wR^4}{2EI}\int_0^{\frac{\pi}{2}} (\sin^2\theta - \cos\theta\sin^2\theta)\,d\theta$$



$$m = -R(1-\cos\theta)$$



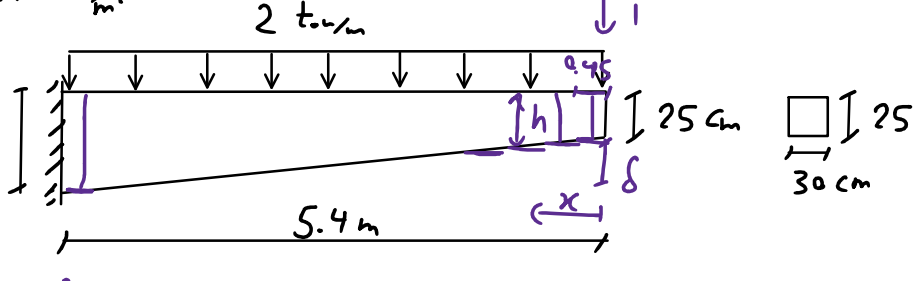
$$\delta_{Bx} = \frac{wR^4}{2EI}\int_0^{\frac{\pi}{2}} \left(\frac{1-\cos 2\theta}{2} - \cos\theta\sin^2\theta\right) d\theta$$

$$\frac{wR^4}{2EI}\left[\frac{\theta}{2} - \frac{1}{4}\sin 2\theta - \frac{\sin^3\theta}{3}\right]_0^{\frac{\pi}{2}} = \frac{wR^4}{2EI}\left[\left(\frac{\pi}{4} - 0 - \frac{1}{3}\right) - 0\right]$$

$$\boxed{\delta_{Bx} = 0.226\frac{wR^4}{EI}}$$

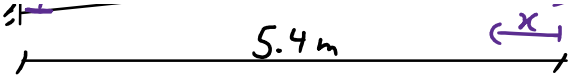
مثال: تغییر مکان سر آزاد تیر را به دست آورید.

$$E = 2\times10^6 \ \frac{ton}{m^2}$$



$$h = 0.25 + \frac{0.45}{5.4}x = 0.25 + \frac{1}{12}x$$

70 cm



$$1 \times \delta = \int \frac{m\,M}{EI}\,dx = \int_{.}^{5.4} \frac{(-x)(-x^2)}{E \times \frac{1}{12} \times 0.3 \times \underbrace{\left(\frac{1}{4} + \frac{1}{12}x\right)^3}_{u}}\,dx$$

$$\begin{cases} u = \frac{1}{4} + \frac{1}{12}x \\ * \quad x = 12\left(u - \frac{1}{4}\right) \\ * \quad dx = 12\,du \end{cases}$$

$$1 \times \delta = \int_{0.25}^{0.7} \frac{12^3 \left(u - \tfrac{1}{4}\right)^3}{2 \times 10^6 \times \frac{0.3}{12}\, u^3}\, 12\,du =$$

$$\delta = \frac{12^5}{2 \times 10^6 \times 0.3} \int_{0.25}^{0.7} \frac{\left(u - \tfrac{1}{4}\right)^3}{u^3}\,du = 0.41472 \int_{0.25}^{0.7} \frac{u^3 - \tfrac{3}{4}u^2 + \tfrac{3}{16}u - \tfrac{1}{64}}{u^3}\,du$$

$$\delta = 0.41472 \int_{0.25}^{0.7} \left(1 - \frac{3}{4}\frac{1}{u} + \frac{3}{16}\frac{1}{u^2} - \frac{1}{64}\frac{1}{u^3}\right) du$$

$u^n \qquad u^{-2}$

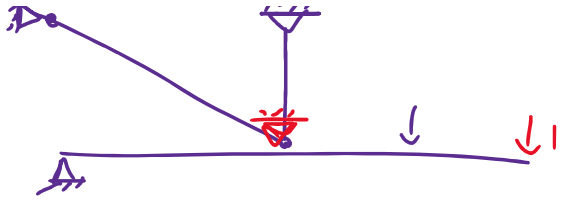
$\frac{1}{n+1}\,u^{n+1}$

$$\delta = 0.41472 \left(u - \frac{3}{4}\ln u + \frac{3}{16}\frac{-1}{u} - \frac{1}{64}\frac{\frac{-1}{2}}{u^2}\right)\Bigg|_{0.25}^{0.7}$$

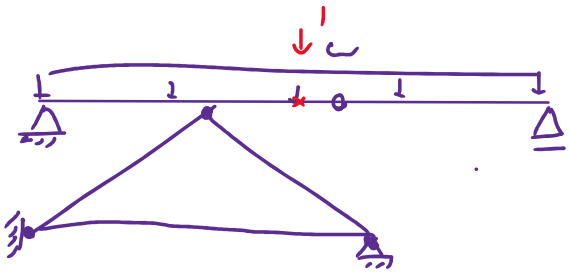
$$\delta = 0.41472 \left[(0.7 - 0.25) - \frac{3}{4}(\ln 0.7 - \ln 0.25) - \frac{3}{16}\left(\frac{1}{0.7} - \frac{1}{0.25}\right) + \frac{1}{128}\left(\frac{1}{0.7^2} - \frac{1}{0.25^2}\right)\right]$$

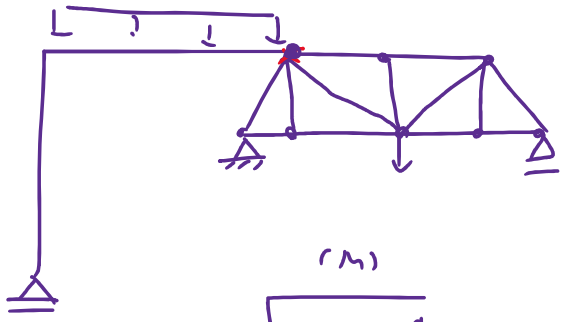
$$\boxed{\delta = 0.021\ m = 2.1\ Cm}$$

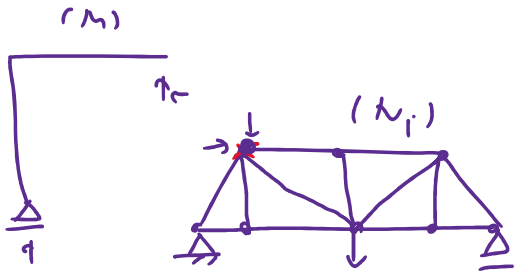
x	h		
0.225	0.26875	0.58682	0.26407
0.675	0.30625	10.7074	4.81832
1.125	0.34375	35.0533	15.774
1.575	0.38125	70.5037	31.7267
2.025	0.41875	113.086	50.8889
2.475	0.45625	159.631	71.8339
2.925	0.49375	207.901	93.5554
3.375	0.53125	256.404	115.382
3.825	0.56875	304.18	136.881
4.275	0.60625	350.633	157.785
4.725	0.64375	395.416	177.937
5.175	0.68125	438.34	197.253
			1054.1
			0.02108



$$1 \times \delta = \int \frac{mM}{EI} dx + \sum \frac{n_i N_i l_i}{EA_i}$$







$$1 \times \delta = \int \frac{mM}{EI} dx + \sum \frac{n_i N_i l_i}{EA_i}$$