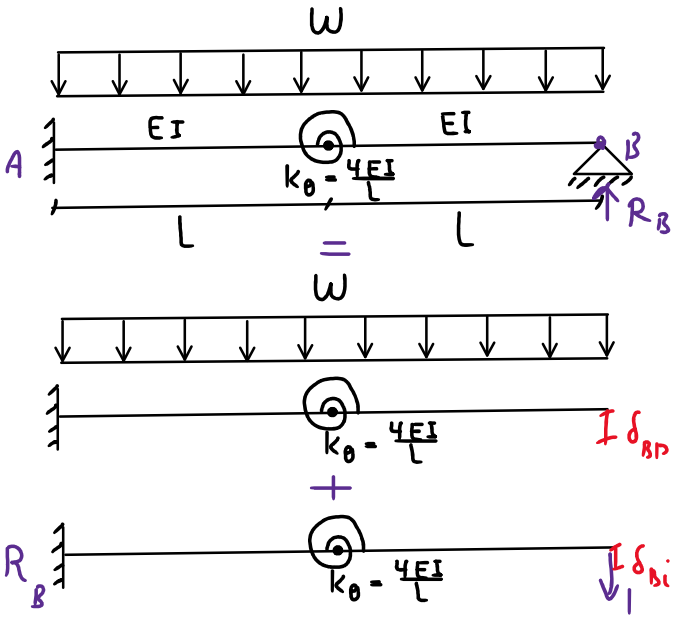
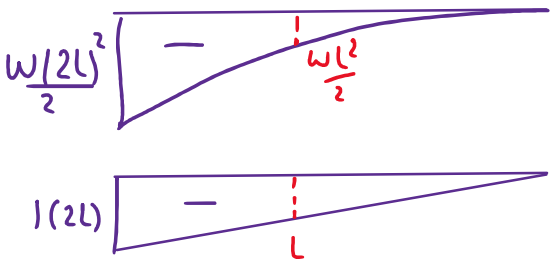
مثال: تیر شکل زیر را به روش‌های مختلف تحلیل کنید.

① سازگاری تغییر شکل‌ها

$\delta_B = 0$



الف) بار واحد



$\delta_{BP} + R_B \delta_{Bi} = 0$

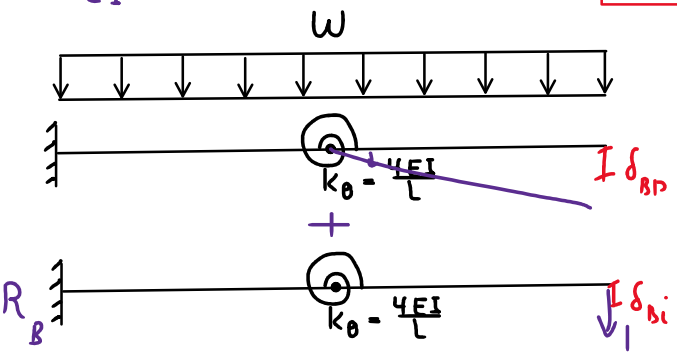
$1 \times \delta_{BP} = \int \frac{m M}{EI} dx + m \frac{M}{k_\theta}$

$1 \times \delta_{BP} = \frac{1}{EI}\left[\frac{2L}{6}\left\{(2WL^2)(-2L) + 4\left(\frac{WL^2}{2}\right)(-L) + 0\right\}\right] + (L)\left(\frac{WL^2}{2k_\theta \to \frac{4EI}{L}}\right) = \frac{17}{8}\frac{WL^4}{EI}$

$1 \times \delta_{Bi} = \frac{1}{EI}\left(\frac{2L}{3}\right)(2L)^2 + \frac{L^2}{K_\theta} = \left(\frac{8}{3} + \frac{1}{4}\right)\frac{L^3}{EI} = \frac{35}{12}\frac{L^3}{EI}$

$\frac{17}{8}\frac{WL^4}{EI} + R_B\left(\frac{35}{12}\frac{L^3}{EI}\right) = 0 \rightarrow$ $\boxed{R_B = \frac{51}{70}WL}$

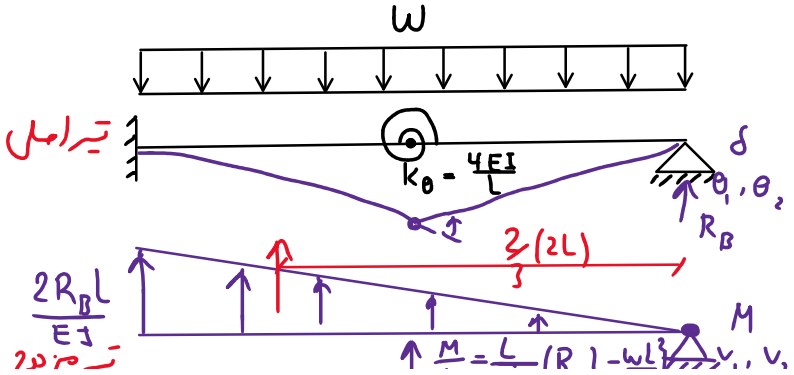
ب) روابط خطی

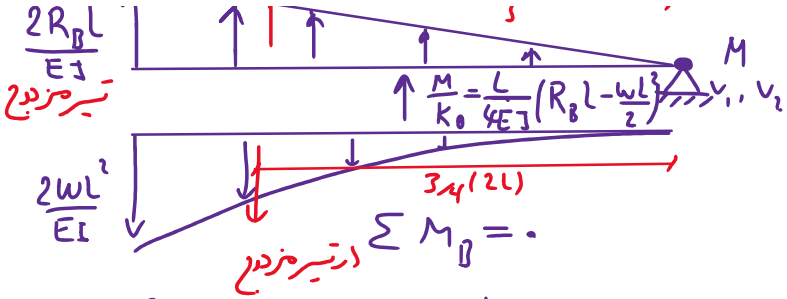


$\delta_{BP} = \frac{W(2L)^4}{8EI} + \frac{WL^2}{2k_\theta}L = \frac{17}{8}\frac{WL^4}{EI}$

$\delta_{Bi} = \frac{1(2L)^3}{3EI} + \frac{L}{k_\theta}(L) = \frac{35}{12}\frac{L^3}{EI}$

② معادلات تعادل تیر مزدوج

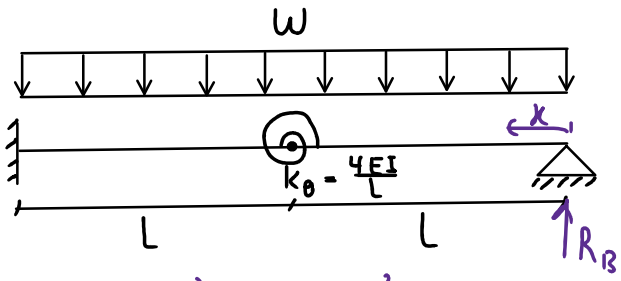




$$\sum M_B = 0$$

$$\frac{1}{2}\left(\frac{2R_B L}{EI}\right)(2L)\left(\frac{2}{3}\times 2L\right) + \frac{L}{4EI}\left(R_B L - \frac{wL^2}{2}\right)(L) - \frac{1}{2}\left(\frac{2wL^2}{EI}\right)(2L)\left(\frac{3}{4}\times 2L\right) = 0$$

$$\frac{8}{3}R_B + \frac{1}{4}R_B - \frac{1}{8}wL - 2wL = 0 \rightarrow \frac{35}{12}R_B = \frac{17}{8}wL \rightarrow \boxed{R_B = \frac{51}{70}wL}$$



② روش کاستیلیانو

$$\delta_B = 0 \rightarrow \frac{\partial U}{\partial R_B} = 0$$

$$U = \frac{1}{2}\int \frac{M^2}{EI}dx + \frac{1}{2}\frac{M^2}{k_\theta}$$

تیر $M = R_B x - \frac{wx^2}{2}$ $\quad \frac{\partial M}{\partial R_B} = x$

فنر $M = R_B L - \frac{wL^2}{2}$ $\quad \frac{\partial M}{\partial R_B} = L$

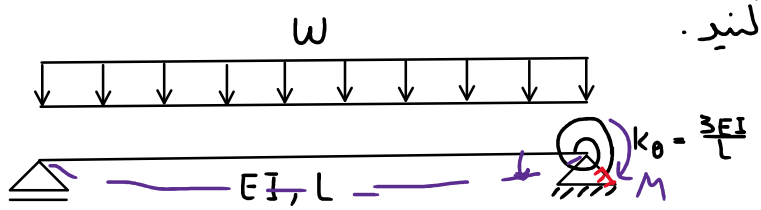
$$\frac{\partial U}{\partial R_B} = \int \frac{M}{EI}\left(\frac{\partial M}{\partial R_B}\right)dx + \frac{M}{k_\theta}\frac{\partial M}{\partial R_B} = 0$$

$$\frac{1}{EI}\int_0^{2L}\left(R_B x - \frac{wx^2}{2}\right)(x)\,dx + \frac{L}{4EI}\left(R_B L - \frac{wL^2}{2}\right)(L) = 0$$

$$\frac{1}{EI}\left(R_B\frac{x^3}{3} - \frac{wx^4}{8}\right)\Big]_0^{2L} + \frac{1}{4}R_B\frac{L^3}{EI} - \frac{1}{8}\frac{wL^4}{EI} = 0 \rightarrow \frac{8}{3}R_B L^3 - 2wL^4 + \frac{1}{4}R_B L^3 - \frac{1}{8}wL^4 = 0$$

$$\boxed{R_B = \frac{51}{70}wL}$$

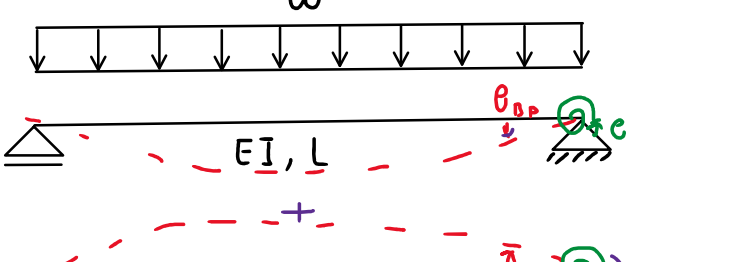
مثال: تیر شکل زیر را به روش های مختلف تحلیل کنید.

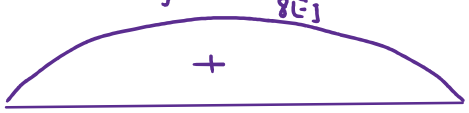


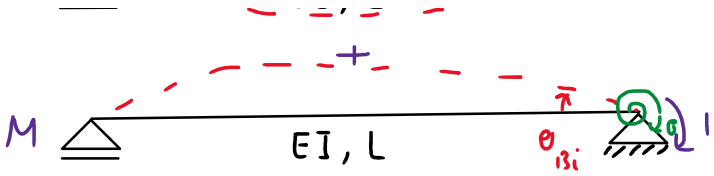
① سازگاری تغییر شکل ها

$\theta = 0$

$\theta_{beam} = \theta_{spring}$







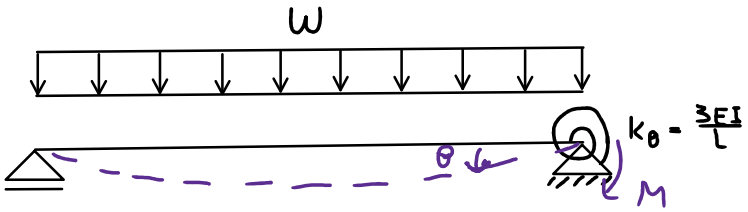
$$1\times\theta_{BP}=\int\frac{mM}{EI}dx=\left(\frac{1}{3}\right)\left(\frac{wL^2}{8EI}\right)(-1)=\frac{-wL^3}{24EI}$$

$$1\times\theta_{Bi}=\int\frac{m^2}{EI}dx=\left(\frac{L}{3}\right)(-1)^2=\frac{L}{3EI}$$

$$-\frac{wL^3}{24EI}+\frac{ML}{3EI}=\frac{-ML}{3EI}\rightarrow\boxed{M=\frac{wL^2}{16}}$$

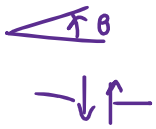
$$-\frac{wL^2}{24EI}+M\left(\frac{L}{3EI}+\frac{1}{\frac{3EI}{L}}\right)=0$$

$$M=\frac{wL^2}{16}$$

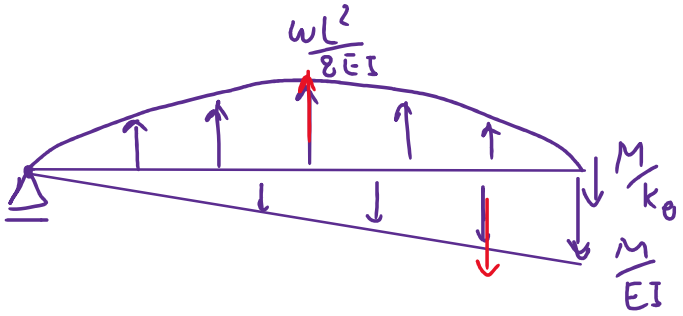


$\delta=0$

$\theta=\frac{M}{k_\theta}$



② تیر مزدوج



$M=0$

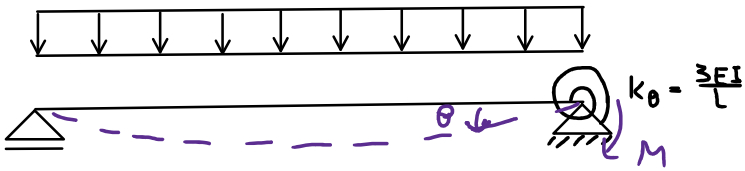
$V=\frac{M}{k_\theta}$

$$\frac{2}{3}\left(\frac{wL^2}{8EI}\right)(L)\left(\frac{L}{2}\right)-\frac{M}{k_\theta}(L)-\frac{1}{2}\left(\frac{M}{EI}\right)(L)\left(\frac{2}{3}L\right)=0$$

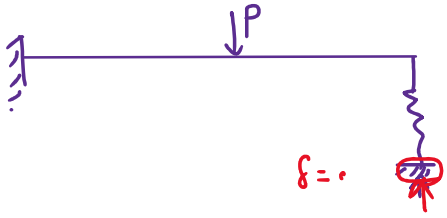
$k_\theta=\frac{3EI}{L}$

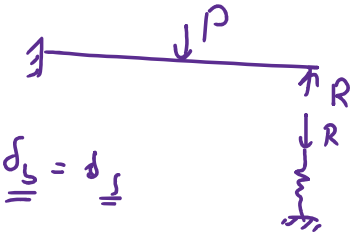
$$\frac{wL^4}{24}-\frac{ML^2}{3}-\frac{ML^2}{3}=0\rightarrow\boxed{M=\frac{wL^2}{16}}$$

② روش توزیع لنگر (کراس)



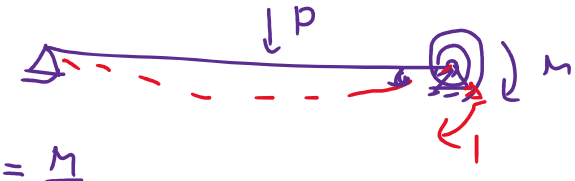
$$M=\frac{wL^2}{8}\times\frac{1}{2}=\frac{wL^2}{16}$$

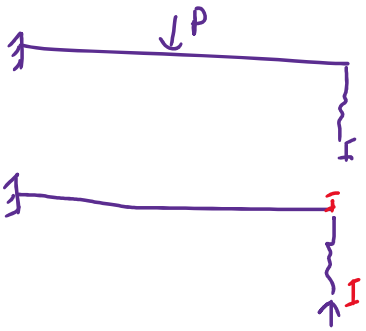




$$-\frac{Pl^3}{3}+\frac{Rl^3}{3}=R$$

$$-\frac{Pl^3}{3EI} \pm \frac{Rl^3}{3EI} = -\frac{R}{k} \qquad R\left(\frac{1}{k_s} + \frac{1}{k_1}\right)$$





$$\frac{Pl^2}{EI} - \frac{Ml}{EI} = \frac{M}{k_\theta}$$