مثال : نیروی فنر را به دست آورید .

$EI = 1000\ t \cdot m^2$

$1200\ kg/m$

$K = 100\ \dfrac{t}{m}$

$3\ m$

$3.6 - F$
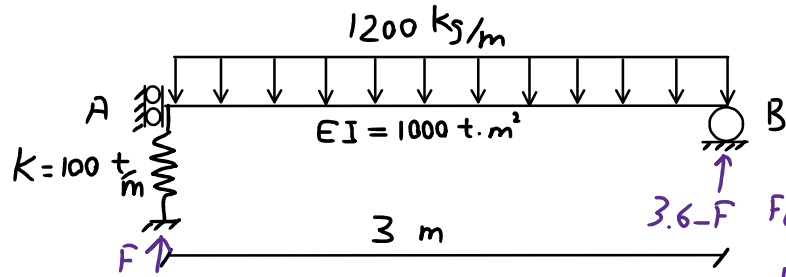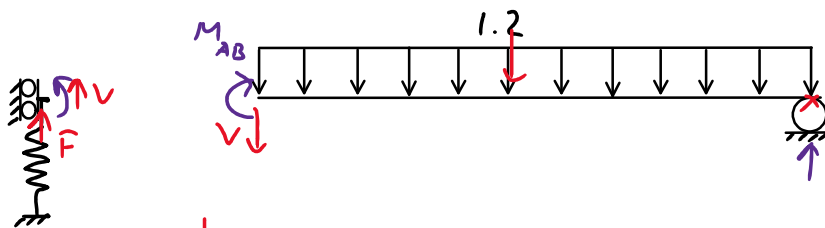
$FEM_{AB} - \frac{1}{2} FEM_{BA} =$

$-\dfrac{wl^2}{12} - \dfrac{1}{2}\dfrac{wl^2}{12} = -\dfrac{3}{2}\dfrac{wl^2}{12_4}$

$$M_{AB} = \frac{3EI}{3}\left(\frac{\Delta}{2}\right) - \frac{1.2 \times 3^2}{8} = \frac{1000}{3}\Delta - 1.35$$

$$(3.6 - F) \times 3 - 3.6 \times \frac{3}{2} = \frac{1000}{3} \times \frac{F}{100} - 1.35$$

$$6.75 = \frac{19}{3}F \quad \rightarrow \quad \boxed{F = 1.066\ ton}$$

روش شیب - افت

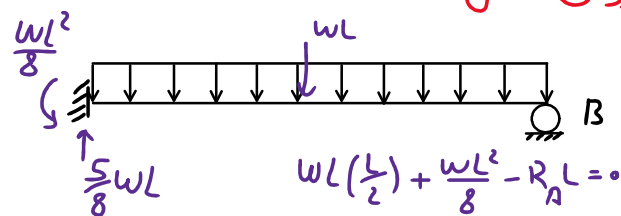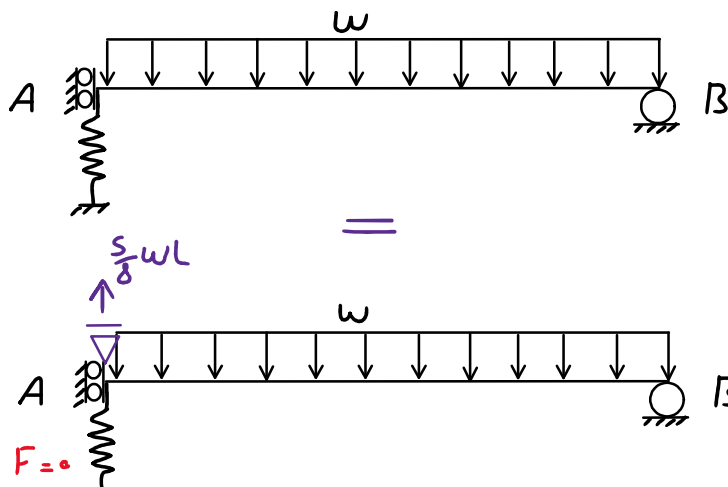

M_{AB}

1.2

V

F

V

Δ : مجهول

معادله : $V + F = 0$

$$V = \frac{1}{3}\left(M_{AB} - 3.6 \times \frac{3}{2}\right) = \frac{1}{3}\left(\frac{1000}{3}\Delta - 1.35 - 5.4\right) = \frac{1000}{9}\Delta - 2.25$$

$$V + F = \frac{1000}{9}\Delta - 2.25 + 100\Delta = 0 \quad \rightarrow \quad \Delta = 0.01066\ m \quad \rightarrow \quad \boxed{F = 1.066\ ton}$$

روش کنتی

w

$\dfrac{wl^2}{8}$

wl

$\dfrac{5}{8}wl$

$wl\left(\frac{l}{2}\right) + \dfrac{wl^2}{8} - R_A l = 0$

$=$

$\dfrac{5}{8}wl$

w

$F = 0$

$F = 0$

$\dfrac{5}{8} wl$

A B

Δ

$\Delta = \dfrac{F}{\Sigma k}$

سفتی موازی

$F = F_1 + F_2$ و ثابت میباشد δ

$F = k_1 \Delta + k_2 \Delta \rightarrow \Delta = \dfrac{F}{\Sigma k} \rightarrow F_1 = \dfrac{k_1}{\Sigma k} F$

$F \rightarrow$

$\Delta = \Delta_1 + \Delta_2$ ن ول مکیا

$F = \dfrac{3EI}{L^3} \Delta$

A B

$\dfrac{3EI}{L^2}\Delta$
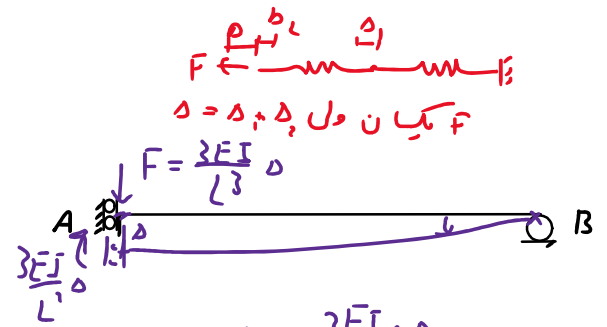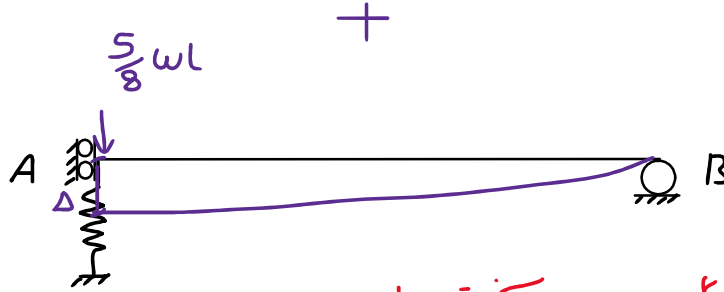
$M_{AB} = \dfrac{3EI}{L}\left(\dfrac{\Delta}{L}\right)$

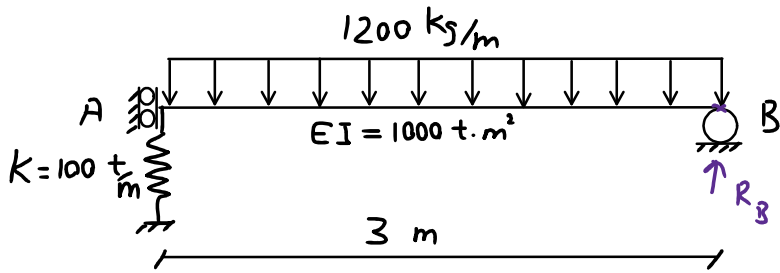$k = \dfrac{3EI}{L^3} = \dfrac{3 \times 1000}{3^3} = \dfrac{1000}{9}$
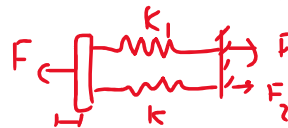
$F = \dfrac{k_s}{k_1 + k_s} \times \dfrac{5}{8}wl = \dfrac{100}{100 + 111.11} \times \dfrac{5}{8} \times 1.2 \times 3 = \boxed{1.068 \; \text{tan}}$
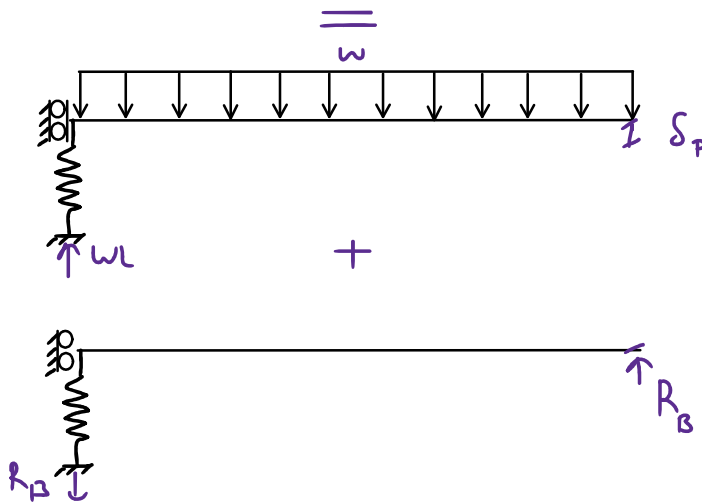

روش سازگاری در تغییر شکلها

$1200 \; \text{kg/m}$

A $EI = 1000 \; t \cdot m^2$ B

$k = 100 \; \dfrac{t}{m}$ $\uparrow R_B$

$3 \; m$

R_B : مجهول

$\Delta_B = 0$: معادله

$\delta_P + R_B \, \delta_i = 0$

w

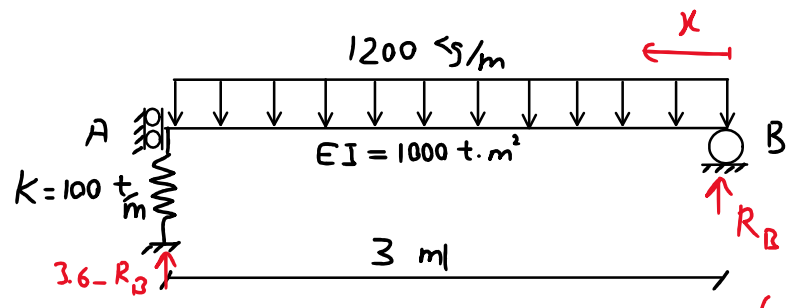$\downarrow \delta_P$

$\uparrow wl$

$+$

$\uparrow R_B$

R_B

$\dfrac{wl}{k} + \dfrac{wL^4}{8EI} = \dfrac{R_B}{k} + \dfrac{R_B L^3}{3EI}$

$\dfrac{1.2 \times 3}{100} + \dfrac{1.2 \times 3^4}{8 \times 1000} = R_B\left(\dfrac{1}{100} + \dfrac{3^3}{3 \times 1000}\right) \rightarrow R_B = 2.534$

$$\boxed{F = 1.066 \ tan}$$

روش کاستیلیانو

$$1200 \ 5/m$$

A

$$EI = 1000 \ t \cdot m^2$$

B

$$k = 100 \ \frac{t}{m}$$

$$3.6 - R_B$$

$$3 \ m|$$

$$\uparrow R_B$$

$$U = \frac{1}{2} \int \frac{M^2}{EI} dx + \frac{1}{2} \frac{F^2}{k}$$

$$\Delta_B = \frac{\partial U}{\partial R_B} = 0$$

$$\begin{cases} M = R_B x - \frac{1.2 x^2}{2} \qquad \frac{\partial M}{\partial R_B} = x \\ F = 3.6 - R_B \qquad \frac{\partial F}{\partial R_B} = -1 \end{cases}$$

$$\int \frac{M}{EI} \left(\frac{\partial M}{\partial R_B} \right) dx + \frac{F}{k} \left(\frac{\partial F}{\partial R_B} \right) = 0$$
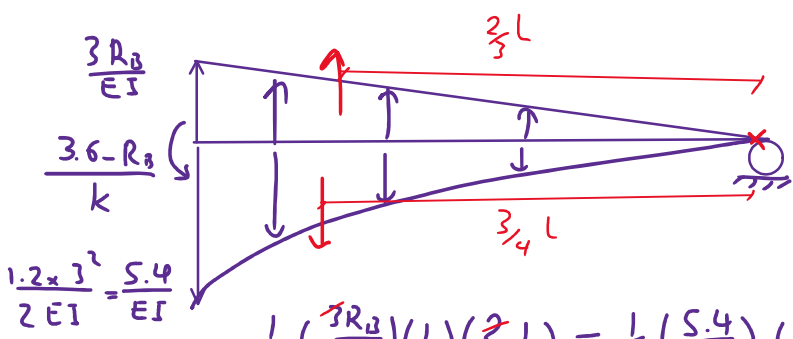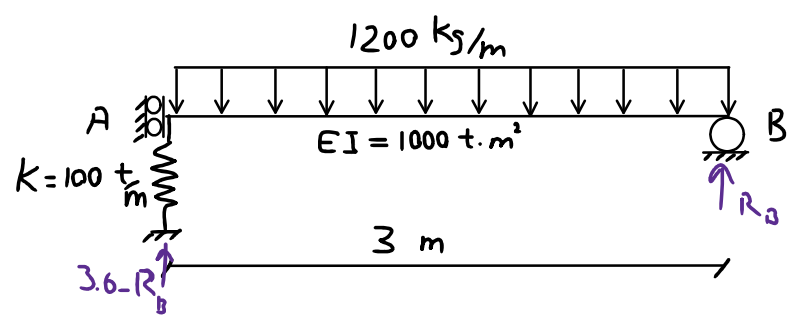
$$\frac{1}{EI} \int_0^3 \left(R_B x - 0.6 x^2 \right) x \, dx + \frac{1}{k} (3.6 - R_B)(-1) = 0$$

$$\frac{1}{1000} \left(R_B \frac{x^3}{3} - \frac{0.6}{4} x^4 \right) \Big|_0^3 + \frac{1}{100} (R_B - 3.6) = 0$$

$$\frac{1}{1000} \left(9 R_B - 12.15 \right) + \frac{1}{100} (R_B - 3.6) = 0$$

$$9 R_B - 12.15 + 10 R_B - 36 = 0 \quad \longrightarrow \quad R_B = 2.534 \quad \longrightarrow \quad \boxed{F = 1.066 \ tan}$$

روش تیر مزدوج

$$1200 \ k5/m$$

A

$$EI = 1000 \ t \cdot m^2$$

B

$$k = 100 \ \frac{t}{m}$$

$$3.6 - R_B$$

$$3 \ m$$

$$\uparrow R_B$$

$$\frac{3 R_B}{EI}$$

$$\frac{2}{3} L$$

$$\frac{3.6 - R_B}{k}$$

$$+ \quad \circlearrowleft \sum M_B = 0$$

$$\frac{3}{4} L$$

$$\frac{1.2 \times 3^2}{2 EI} = \frac{5.4}{EI}$$

$$\frac{1}{2} \left(\frac{3 R_B}{EI} \right) (L) \left(\frac{2}{3} L \right) - \frac{1}{3} \left(\frac{5.4}{EI} \right) (L) \left(\frac{3}{4} L \right) - \frac{3.6 - R_B}{0} = 0$$

$$\frac{1.2 \times 3^2}{2EI} = \frac{5.4}{EI} \quad \checkmark$$

$$\frac{1}{2}\left(\frac{3R_B}{EI}\right)(L)\left(\frac{2}{3}L\right) - \frac{1}{3}\left(\frac{5.4}{EI}\right)(L)\left(\frac{3}{4}L\right) - \frac{3.6 - R_B}{k} = 0$$

$$\frac{1}{1000}\left(3^2 R_B - 12.15\right) - \frac{1}{100}\left(3.6 - R_B\right) = 0 \quad \rightarrow \quad R_B = 2.534$$

$$\boxed{\bar{F} = 1.066 \ \text{ton}}$$