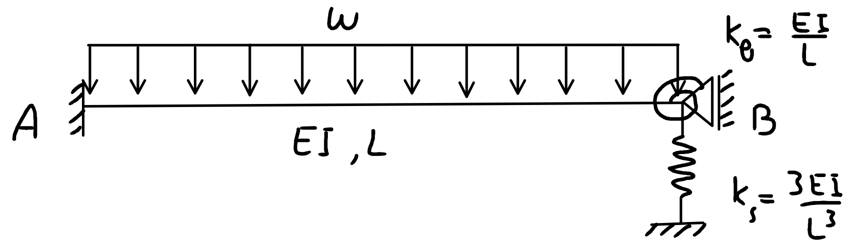
Slope-Deflection 11

Wednesday, March 27, 2024 16:11

مثال : نیروهای فنرها را به دست آورید .



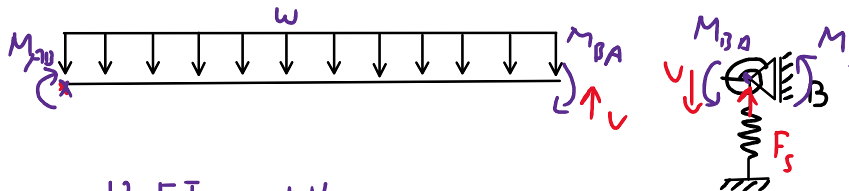
1 روش شیب ـ افت :

مجهولات : θ_B , Δ

معادلات : $\Sigma M_B = 0$, معادله برش

$$M_{AB} = \frac{2EI}{L}\left(\theta_B - 3\frac{\Delta}{L}\right) - \frac{wL^2}{12}$$

$$M_{BA} = \frac{2EI}{L}\left(2\theta_B - 3\frac{\Delta}{L}\right) + \frac{wL^2}{12}$$



$$V = \frac{1}{L}(M_{AB} + M_{BA}) + \frac{wL}{2} = \frac{6EI}{L^2}\theta_B - 12\frac{EI}{L^3}\Delta + \frac{wL}{2}$$

$$\text{①}\ M_{BA} + \underbrace{M_s}_{k_\theta \theta_B} = 0 \rightarrow \frac{5EI}{L}\theta_B - \frac{6EI}{L^2}\Delta + \frac{wL^2}{12} = 0 \quad \times \frac{L}{EI}$$

$$\text{②}\ V - \underbrace{F_s}_{k_s \Delta} = 0 \rightarrow \frac{6EI}{L^2}\theta_B - 15\frac{EI}{L^3}\Delta + \frac{wL}{2} = 0 \quad \times \frac{L^2}{EI}$$

$$6 \quad 5\cancel{\theta_B} - 6\frac{\Delta}{L} = -\frac{wL^3}{12EI}$$

$$-5 \quad 6\cancel{\theta_B} - 15\frac{\Delta}{L} = -\frac{wL^3}{2EI}$$

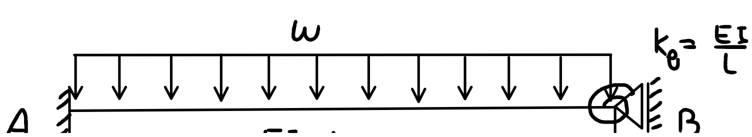
$$(75 - 36)\frac{\Delta}{L} = \left(\frac{5}{2} - \frac{1}{2}\right)\frac{wL^3}{EI} \rightarrow \boxed{\frac{\Delta}{L} = \frac{2}{39}\frac{wL^3}{EI}}$$

$$6\theta_B = \cancel{15}^{5} \times \frac{2}{\cancel{39}_{13}}\frac{wL^3}{EI} - \frac{1}{2}\frac{wL^3}{EI} = \frac{20-13}{26}\frac{wL^3}{EI} \rightarrow \boxed{\theta_B = \frac{7}{156}\frac{wL^3}{EI}}$$

$$M_s = k_\theta \theta_B = \frac{EI}{L} \times \frac{7}{156}\frac{wL^3}{EI} \rightarrow \boxed{M_s = \frac{7}{156}wL^2}$$

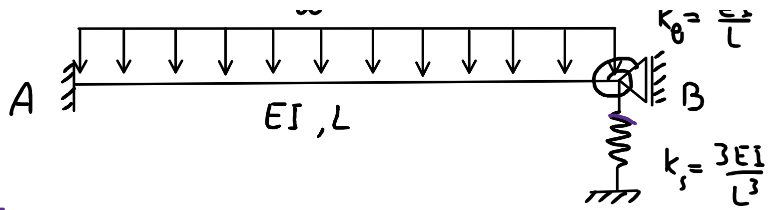
$$F_s = k_s \Delta = \frac{3EI}{L^3} \times \frac{2}{39}\frac{wL^4}{EI} \rightarrow \boxed{F_s = \frac{2}{13}wL}$$

2 روش تیر مزدوج



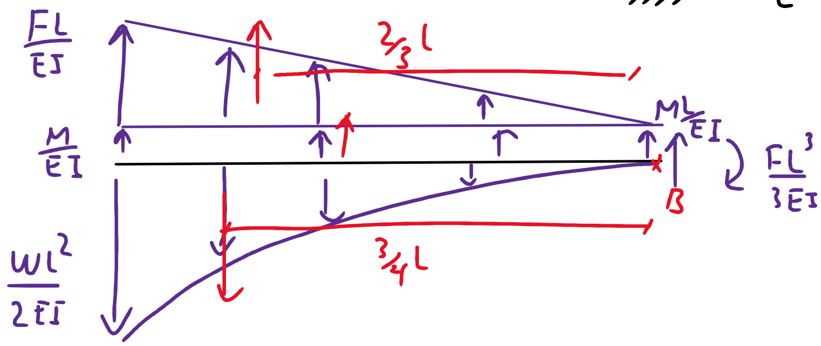
$$\theta = \frac{M}{k_\theta} = \frac{ML}{EI} \rightarrow$$

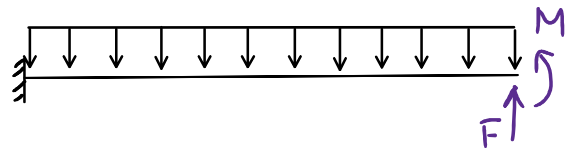
(۲) روش تیر مزدوج



$$\theta = \frac{M}{k_\theta} = \frac{ML}{EI} \rightarrow V$$

$$\Delta = \frac{F}{k_s} = \frac{FL^3}{3EI} \rightarrow M$$





$$\overset{+}{\uparrow} \sum F_y = 0 \quad \frac{1}{2}\left(\frac{FL}{EI}\right)(L) + \left(\frac{M}{EI}\right)(L) - \frac{1}{3}\left(\frac{wL^2}{2EI}\right)(L) + \frac{ML}{EI} = 0 \qquad \times \frac{EI}{L}$$

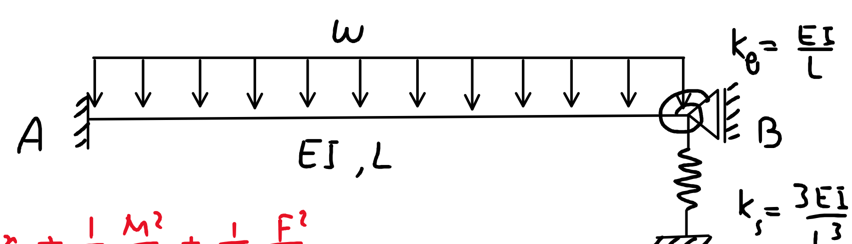
$$\overset{+}{\circlearrowleft} \sum M_B = 0 \quad \frac{FL^2}{2EI}\left(\frac{2}{3}L\right) + \frac{ML}{EI}\left(\frac{L}{2}\right) - \frac{wL^3}{6EI}\left(\frac{3}{4}L\right) + \frac{FL^3}{3EI} = 0 \qquad \times \frac{EI}{L^2}$$

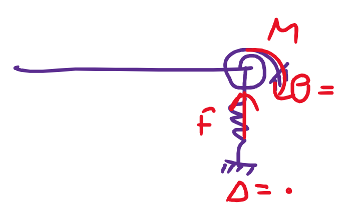
$$\begin{cases} 2M + \frac{1}{2}FL = \frac{1}{6}wL^2 \\ -4 \quad \frac{M}{2} + \frac{2}{3}FL = \frac{1}{8}wL^2 \end{cases}$$

$$\left(\frac{1}{2} - \frac{8}{3}\right)FL = \left(\frac{1}{6} - \frac{1}{2}\right)wL^2 \rightarrow \left(\frac{3-16}{6}\right)FL = -\frac{1}{3}wL^2 \rightarrow \boxed{F = \frac{2}{13}wL}$$

$$2M = -\frac{1}{2} \times \frac{2}{13}wL^2 + \frac{1}{6}wL^2 = \frac{-6+13}{78}wL^2 = \frac{7}{78}wL^2 \rightarrow \boxed{M = \frac{7}{156}wL^2}$$

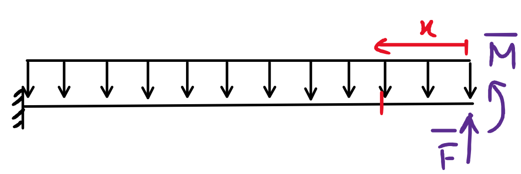
(۳) روش کاستیلیانو





$$U = \frac{1}{2}\int \frac{m^2}{EI}dx + \frac{1}{2}\frac{M^2}{k_\theta} + \frac{1}{2}\frac{F^2}{k_s}$$

$$\frac{\partial U}{\partial \bar{M}} = 0 \quad , \quad \frac{\partial U}{\partial \bar{F}} = 0$$



$$m = \bar{M} + \bar{F}x - \frac{wx^2}{2} \quad ; \quad \frac{\partial m}{\partial \bar{M}} = 1 \quad ; \quad \frac{\partial m}{\partial \bar{F}} = x$$

$$M = \bar{M} \quad ; \quad \frac{\partial M}{\partial \bar{M}} = 1 \quad ; \quad \frac{\partial M}{\partial \bar{F}} = 0$$

$$M = \bar{M} \quad ; \quad \frac{\partial M}{\partial \bar{M}} = 1 \quad ; \quad \frac{\partial M}{\partial \bar{F}} = 0$$

$$F = \bar{F} \quad ; \quad \frac{\partial F}{\partial \bar{M}} = 0 \quad ; \quad \frac{\partial F}{\partial \bar{F}} = 1$$

$$\frac{\partial U}{\partial \bar{M}} = \int \frac{m}{EI} \frac{\partial m}{\partial \bar{M}} dx + \frac{M}{k_\theta} \frac{\partial M}{\partial \bar{M}} + \frac{F}{k_s} \frac{\partial F}{\partial \bar{M}} = 0$$

$$\frac{1}{EI} \int_0^L \left(\bar{M} + \bar{F}x - \frac{wx^2}{2} \right)(1)\, dx + \left(\frac{\bar{M}L}{EI} \right)(1) + 0 = 0$$

$$\frac{1}{EI}\left(\bar{M}x + \bar{F}\frac{x^2}{2} - \frac{wx^3}{6} \right)\Big|_0^L + \frac{\bar{M}L}{EI} = 0 \rightarrow (1)\; 2\frac{\bar{M}L}{EI} + \frac{1}{2}\frac{\bar{F}L^2}{EI} - \frac{wL^3}{6EI} = 0 \quad \times \frac{EI}{L}$$

$$\frac{\partial U}{\partial \bar{F}} = \int \frac{m}{EI} \frac{\partial m}{\partial \bar{F}} dx + \frac{M}{k_\theta} \frac{\partial M}{\partial \bar{F}} + \frac{F}{k_s} \frac{\partial F}{\partial \bar{F}} = 0$$

$$\frac{1}{EI} \int_0^L \left(\bar{M} + \bar{F}x - \frac{wx^2}{2} \right)(x)\, dx + 0 + \left(\frac{\bar{F}L^3}{3EI} \right)(1) = 0$$

$$\frac{1}{EI}\left(\bar{M}\frac{x^2}{2} + \bar{F}\frac{x^3}{3} - \frac{wx^4}{8EI} \right)\Big|_0^L + \frac{\bar{F}L^3}{3EI} = 0 \rightarrow (2)\; \frac{1}{2}\frac{\bar{M}L^2}{EI} + \frac{2}{3}\frac{\bar{F}L^3}{EI} - \frac{wL^4}{8EI} = 0 \quad \times \frac{EI}{L^2}$$

$$\begin{cases} (1)\;\; 2\bar{M} + \frac{1}{2}\bar{F}L = \frac{1}{6}wL^2 & \rightarrow \\ (2)\;\; \frac{1}{2}\bar{M} + \frac{2}{3}\bar{F}L = \frac{1}{8}wL^2 & \rightarrow \end{cases} \quad \boxed{\begin{aligned} \bar{F} &= \frac{2}{13}wL \\ \bar{M} &= \frac{7}{156}wL^2 \end{aligned}}$$