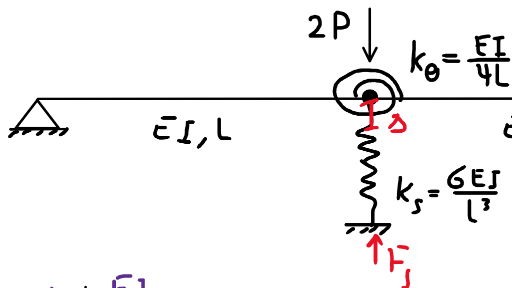
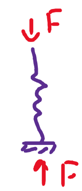
تحلیل سازه های متقارن

مثال : نیروی فنرها را به دست آورید.



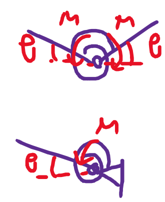
$F_s = k_s \Delta$

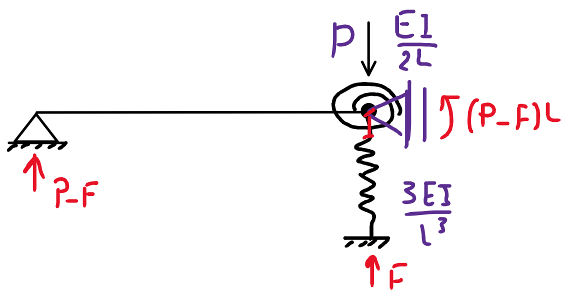
$F_s/2 = \frac{k_s}{2} \Delta$

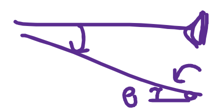


$M = k_\theta \, 2\theta$

$M = 2k_\theta \, \theta$







روش شیب ـ افت

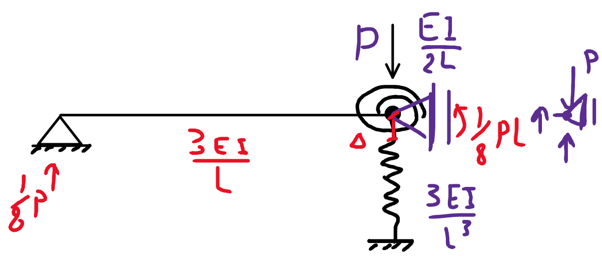
$$M = \frac{3EI}{L}\left(\theta - \frac{\Delta}{L}\right)$$

$$-(P-F)\cancel{L} = \frac{3\cancel{EI}}{\cancel{L}}\left(\frac{(P-F)\cancel{L}}{\frac{\cancel{EI}}{2\cancel{L}}} - \frac{F}{\frac{3\cancel{EI}}{\cancel{L^3}} \times \cancel{L}}\right) \rightarrow F-P = 3\left(2(P-F) - \frac{F}{3}\right)$$

$$F-P = 6P - 7F$$

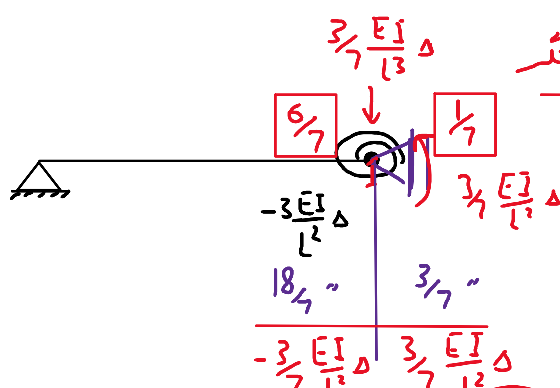
$\boxed{F_s = \frac{7}{8}P}$, $\boxed{M_s = \frac{1}{8}PL}$

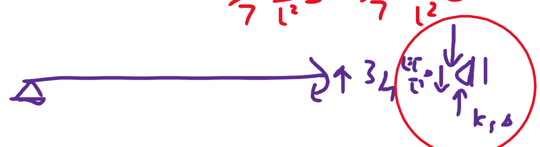
روش توزیع لنگر



$$\frac{3}{7}\frac{EI}{L^3}\Delta + \frac{3EI}{L^3}\Delta = P \rightarrow \Delta = \frac{7}{24}\frac{PL^3}{EI}$$

$$F_s = \frac{3EI}{L^3} \times \frac{7}{24}\frac{PL^3}{EI} = \boxed{\frac{7}{8}P}$$

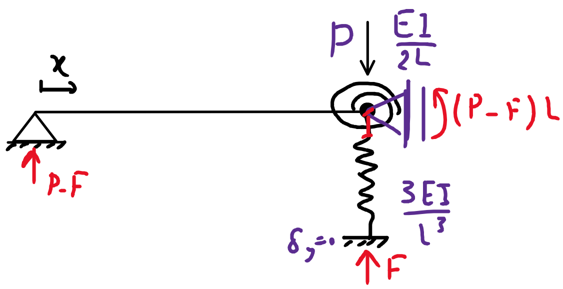




$\Sigma F_y = 0$

$\boxed{F_s = \frac{7}{8}P} \leftarrow \frac{3EI}{L^3} , \frac{3}{7}\frac{EI}{L^3}$: مفهوم سختی

روش کاستیلیانو



$$U = \frac{1}{2}\int \frac{M^2}{EI}dx + \frac{1}{2}\frac{N^2}{k_s} + \frac{1}{2}\frac{M^2}{k_\theta}$$

$$\frac{\partial U}{\partial F} = \int \frac{M}{EI}\left(\frac{\partial M}{\partial F}\right)dx + \frac{N}{k_s}\left(\frac{\partial N}{\partial F}\right) + \frac{M}{k_\theta}\left(\frac{\partial M}{\partial F}\right) = 0$$

$\frac{1}{EI}\int_0^L (F-P)x^2 dx + \frac{F}{\ldots} + \frac{(F-P)L^2}{\ldots}$ ... (1) $M = (P-F)x$ $\frac{\partial M}{\partial F} = -x$

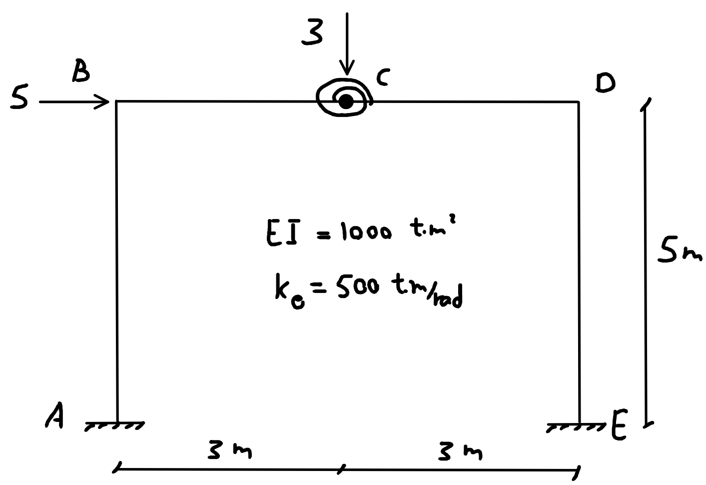
$$\frac{1}{EI}\int_0^L (F-P)x^2\,dx + \frac{F}{k_s} + \frac{(F-P)L^2}{k_\theta} = 0$$

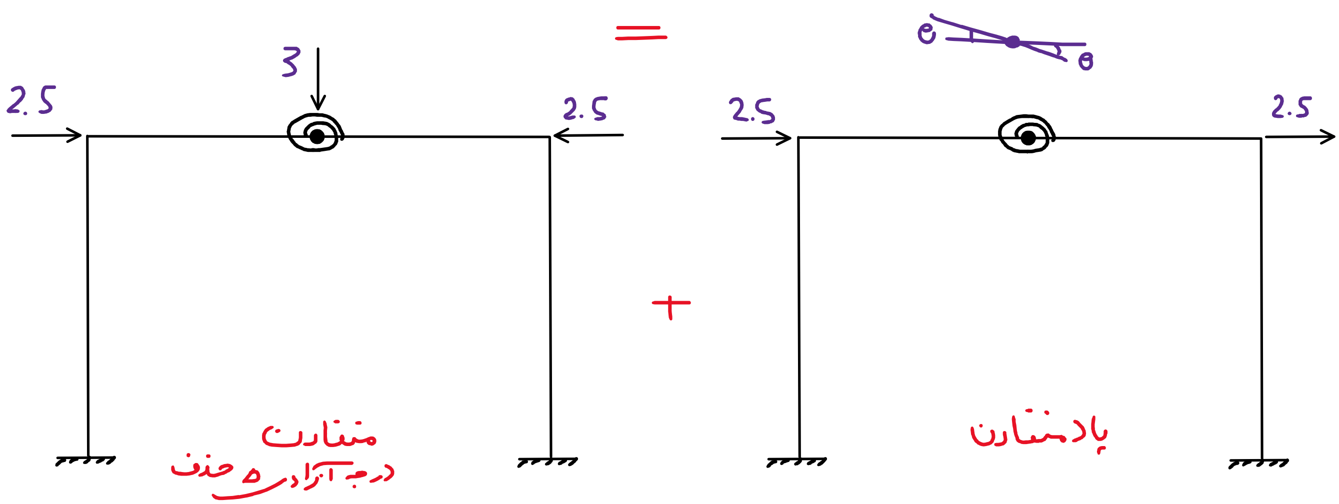
$$\frac{1}{EI}(F-P)\frac{x^3}{3}\Big|_0^L + \frac{FL^3}{3\cancel{EI}} + (F-P)\frac{2L^3}{\cancel{EI}} = 0$$

$$\boxed{F = \frac{7}{8}P}$$

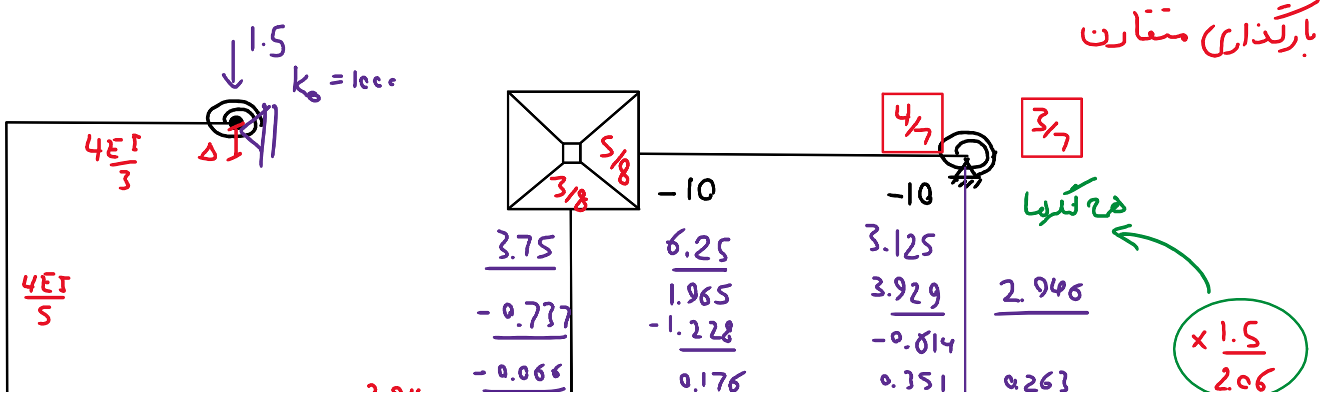
$$\begin{cases} ① \; M = (P-F)x \,, & \frac{\partial M}{\partial F} = -x \\ ② \; N = F \,, & \frac{\partial N}{\partial F} = 1 \\ ③ \; M = (P-F)L \,, & \frac{\partial M}{\partial F} = -L \end{cases}$$

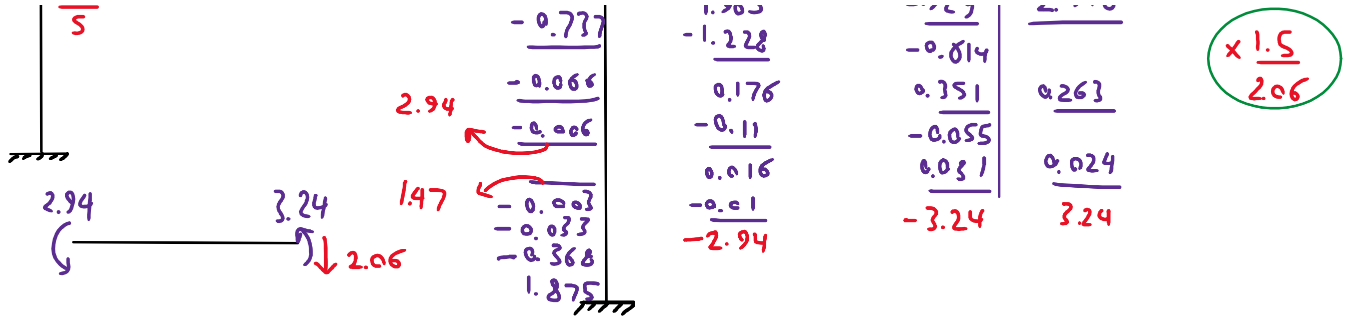
مثال: لنگرهای انتهایی اعضا را به دست آورید.



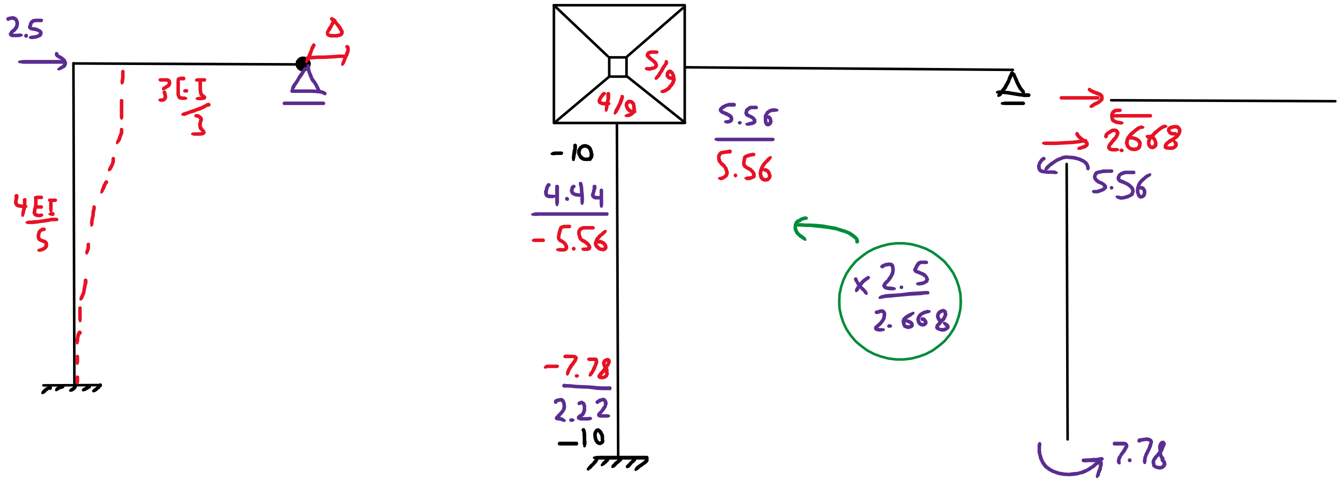


حل بارگذاری متقارن

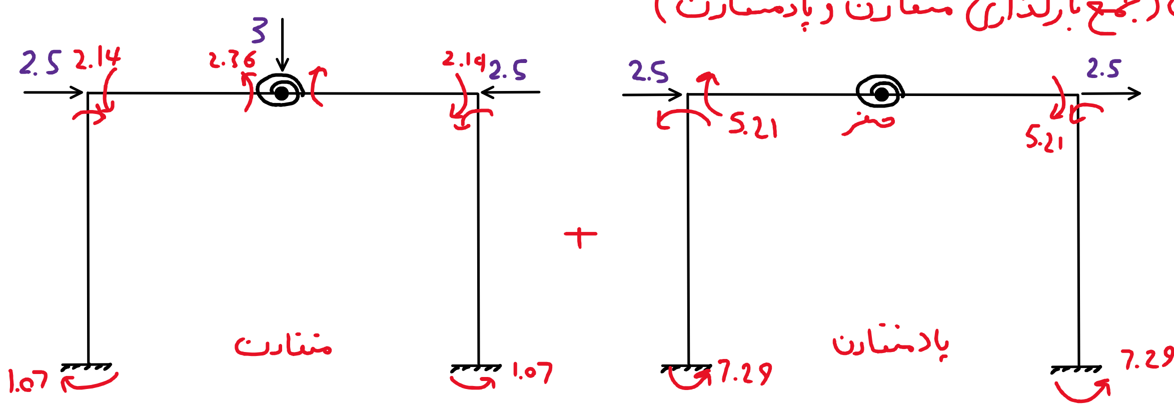


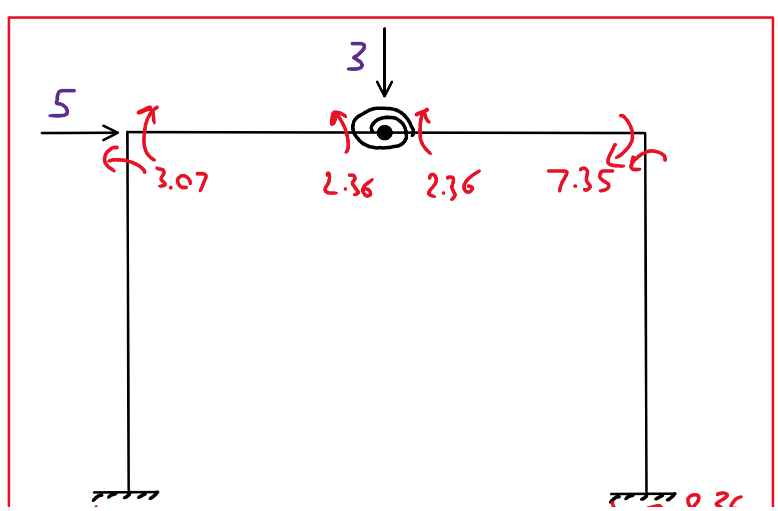


حل بارگذاری پادمتقارن



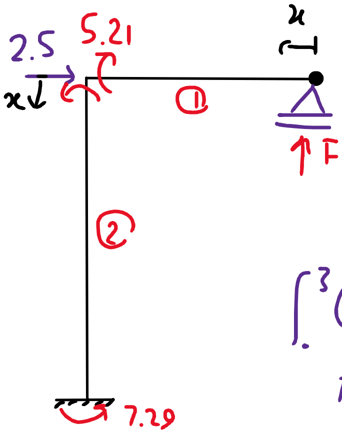
بارگذاری کل (جمع بارگذاری متقارن و پادمتقارن)





روش دیگر (روش کاستیلیانو)

حل بارگذاری پادمتقارن



$$\frac{\partial U}{\partial F} = \int \frac{M}{EI}\left(\frac{\partial M}{\partial F}\right) dx = 0$$

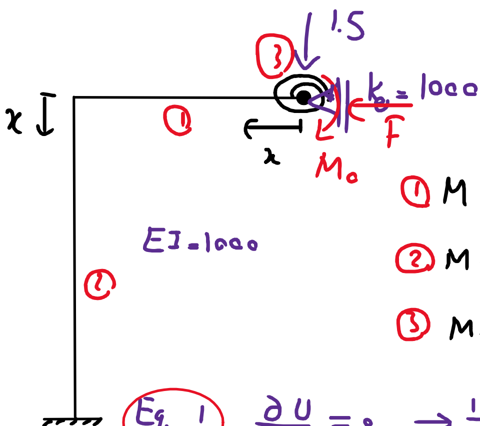
$$\begin{cases} ① \; M = Fx & \frac{\partial M}{\partial F} = x \\ ② \; M = 3F - 2.5x & \frac{\partial M}{\partial F} = 3 \end{cases}$$

$$\int_0^3 (Fx)(x)\,dx + \int_0^5 (3F - 2.5x)(3)\,dx = 0$$

$$\frac{Fx^3}{3}\Big|_0^3 + \left(9Fx - \frac{7.5}{2}x^2\right)\Big|_0^5 = 0 \longrightarrow 9F + 45F - 93.75 = 0$$

$$\boxed{F = 1.736}$$

حل بارگذاری متقارن



$$U = \frac{1}{2}\int \frac{M^2}{EI}dx + \frac{1}{2}\frac{M^2}{k_\theta}$$

$$\begin{cases} \delta_x = 0 \rightarrow \frac{\partial U}{\partial F} = 0 \\ \theta = 0 \rightarrow \frac{\partial U}{\partial M_0} = 0 \end{cases}$$

		$\frac{\partial M}{\partial M_0}$	$\frac{\partial M}{\partial F}$
①	$M = -M_0 - 1.5x$	$= -1$	$= 0$
②	$M = -M_0 - 4.5 + Fx$	$= -1$	$= x$
③	$M = M_0$	$= 1$	0

Eq. 1 $\quad \frac{\partial U}{\partial M_0} = 0 \rightarrow \frac{1}{\cancel{EI}}\int_0^3 (M_0 + 1.5x)\,dx + \frac{1}{\cancel{EI}}\int_0^5 (M_0 + 4.5 - Fx)\,dx + \frac{M_0}{\cancel{k_\theta}} = 0$

$$\left(M_0 x + 1.5\frac{x^2}{2}\right)\Big|_0^3 + \left(M_0 x + 4.5x - F\frac{x^2}{2}\right)\Big|_0^5 + M_0 = 0$$

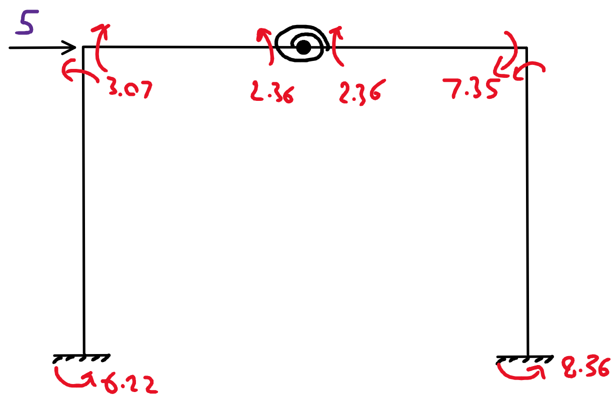
$$3M_0 + 6.75 + 5M_0 + 22.5 - 12.5F + M_0 = 0 \longrightarrow * \; 9M_0 - 12.5F = -29.25$$

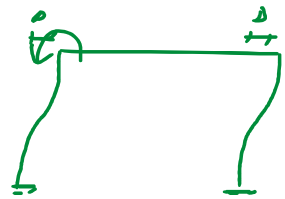
Eq. 2 $\quad \frac{\partial U}{\partial F} = 0 \rightarrow 0 + \frac{1}{EI}\int_0^5 (-M_0 x - 4.5x + Fx^2)\,dx + 0 = 0$

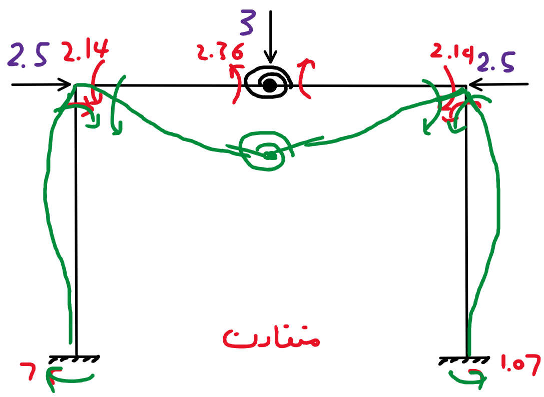
$$\left(-M_0\frac{x^2}{2} - \frac{4.5}{2}x^2 + F\frac{x^3}{3}\right)\Big|_0^5 = 0 \rightarrow * \; -12.5M_0 + 41.67F = 56.25$$

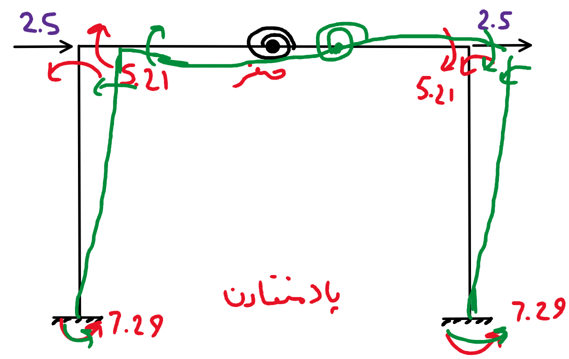
$$\begin{array}{r} 12.5 \\ 9 \end{array}\begin{cases} 9\cancel{M_0} - 12.5F = -29.25 \\ -12.5\cancel{M_0} + 41.67F = 56.25 \end{cases}$$

$$218.78F = 140.625 \rightarrow \boxed{\begin{array}{l} F = 0.643 \\ M_0 = -2.36 \end{array}}$$


5
3.07
2.36
2.36
7.35
8.22
8.36




3
2.5
2.14
2.36
2.14
2.5
متقارن
7
1.07


2.5
2.5
5.21
صفر
5.21
پادمتقارن
7.29
7.29