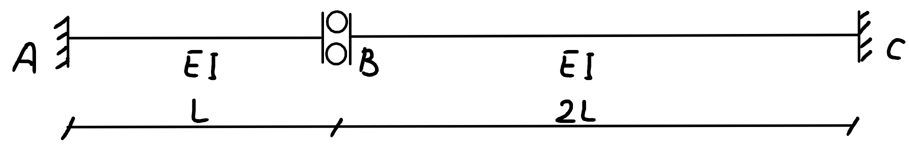
Influence Line 5

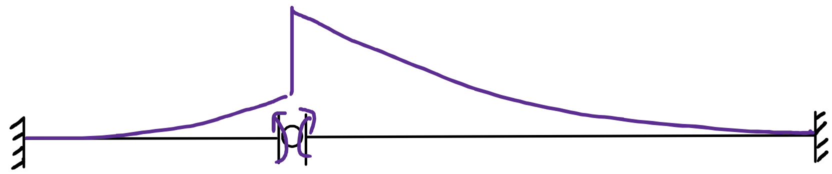
Thursday, June 13, 2024 18:56

خط تأثیر سازه‌های نامعین

مثال: حداکثر لنگر نقاط A ، B و C در اثر عبور بار متمرکز P از روی تیر شکل زیر چقدر است؟

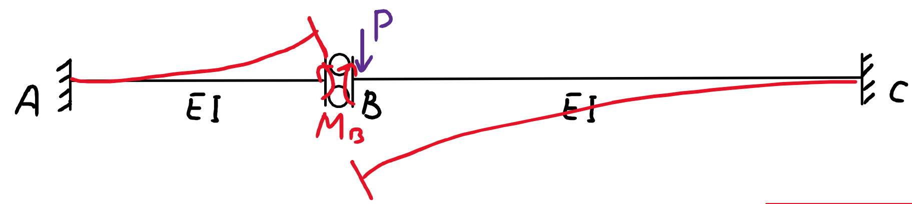


(M_B)



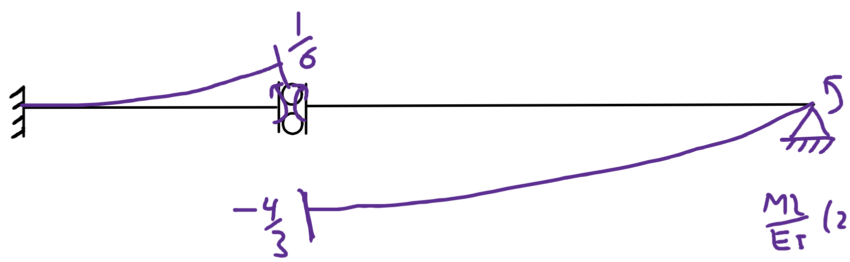
مجهول M_B

معادله $\theta_{BL} = \theta_{BR}$

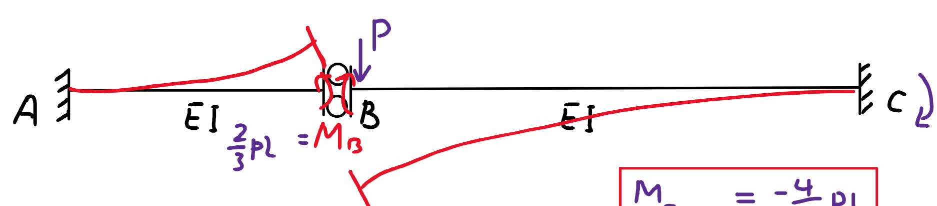


$$\frac{M_B L}{EI} = \frac{P(2L)^2}{2EI} - \frac{M_B(2L)}{EI} \rightarrow 3M_B = 2PL \rightarrow \boxed{M_{B,max} = \frac{2}{3}PL}$$

(M_C)

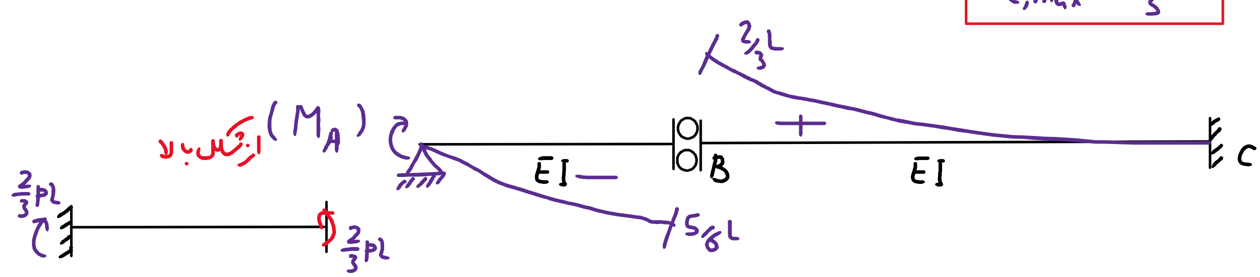


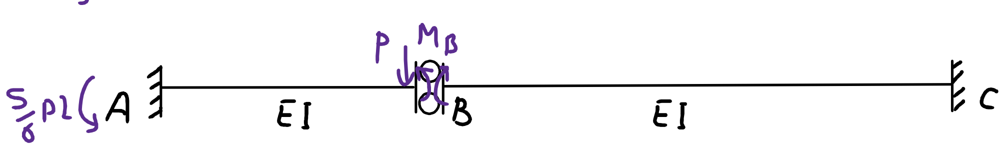
$$\frac{ML}{EI}(2L) + \frac{M(2L)^2}{2EI} = 4\frac{ML^2}{EI}$$



$-2PL + \frac{2}{3}PL$

$$\boxed{M_{C,max} = -\frac{4}{3}PL}$$



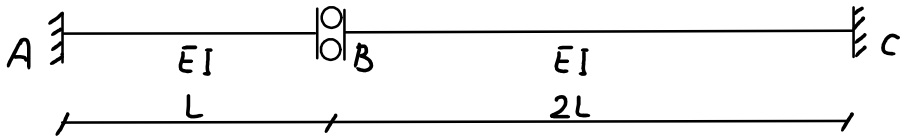


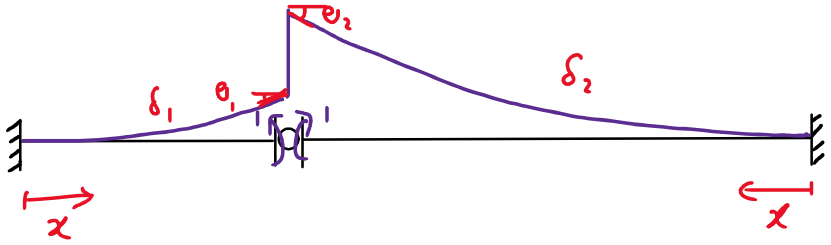
$$\frac{PL^2}{2EI} - \frac{M_B L}{EI} = \frac{M_B(2L)}{EI} \rightarrow 3M_B = \frac{PL}{2} \rightarrow M_B = \frac{PL}{6}$$

$$\frac{PL^2}{2EI} - \frac{M_B L}{EI} = \frac{M_B (2L)}{EI} \rightarrow 3M_B = \frac{PL}{2} \rightarrow M_B = \frac{PL}{6}$$

$$\boxed{M_{A,max} = -\frac{5}{6}PL}$$

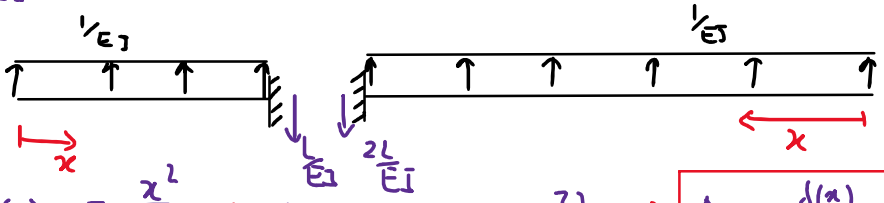
مثال: در مثال فوق، تابع خط تأثیر M_A و M_B را به دست آورید.





$$\begin{cases} \delta_1(x) = 1 \times \frac{x^2}{2EI} \\ \delta_2(x) = 1 \times \frac{x^2}{2EI} \end{cases} \rightarrow \Delta\theta = \frac{L}{EI} + \frac{(2L)}{EI} = \frac{3L}{EI} \rightarrow \boxed{M_B = \frac{\delta(x)}{\Delta\theta} = \frac{x^2}{6L}}$$

کنسول در تیر مزدوج



$$\delta(x)_{\text{اصلی}} = M(x)_{\text{تیر مزدوج}} = \frac{x^2}{2EI} \rightarrow \Delta\theta_{\text{اصلی}} = \Delta V_{\text{تیر مزدوج}} = \frac{3L}{EI} \rightarrow \boxed{M_B = \frac{\delta(x)}{\Delta\theta} = \frac{x^2}{6L}}$$

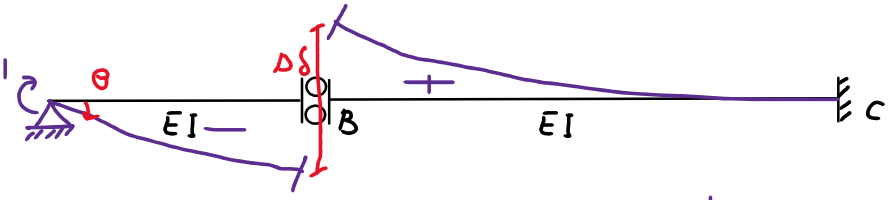
از معادله تعادل و M_B در بخش قبل

بار روی BC $\rightarrow M_A = M_B = \boxed{\frac{x^2}{6L} \quad BC}$

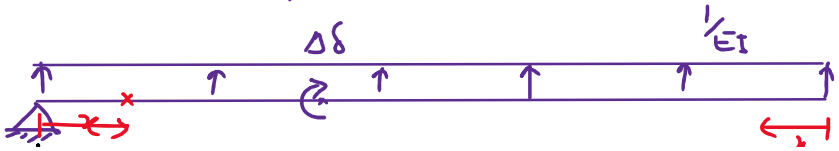
بار روی AB $\rightarrow M_A = M_B - 1 \times x = \boxed{\frac{x^2}{6L} - x \quad AB}$

$\Delta\delta \rightarrow \Delta M$

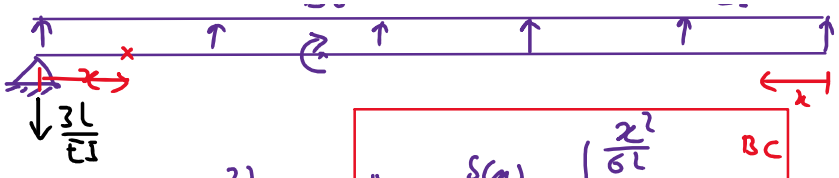
$\theta \rightarrow V$



تیر مزدوج



تیر مزدوج



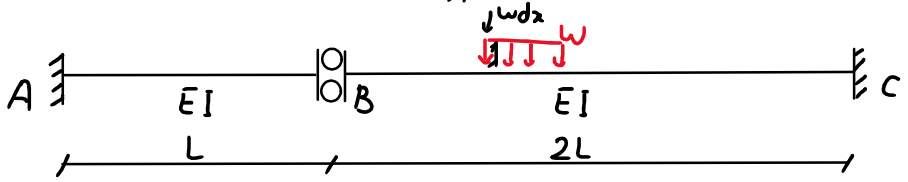
$\delta_2(x) = \frac{x^2}{2EI}$

$\delta_1(x) = \frac{-3L}{EI}x + \frac{x^2}{2EI}$

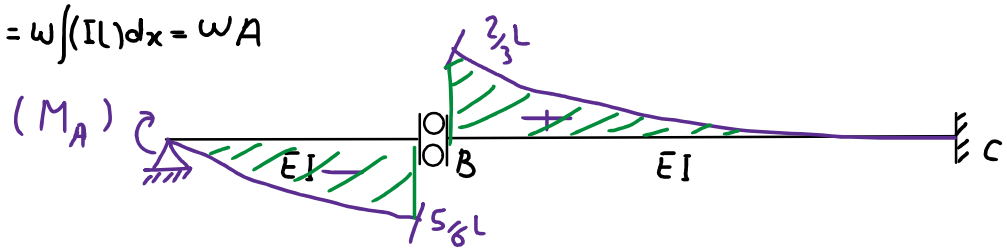
$\rightarrow \theta = \frac{2L}{EI} \rightarrow$

$$M_A = \frac{\delta(m)}{\theta} = \begin{cases} \frac{x^2}{6L} & BC \\ -x + \frac{x^2}{6L} & AB \end{cases}$$

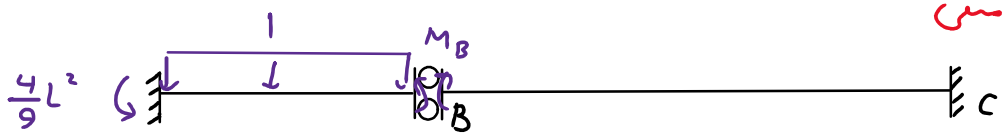
مثال: در مثال فوق، مساحت زیر منحنی خط تاثیر M_A را به دست آورید.



$$M_A = \int (IL)\, w\, dx = w \int (IL)\, dx = wA$$



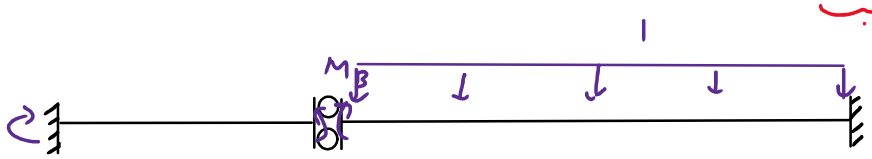
مساحت ناحیه منفی



$$\theta_{BL} = \theta_{BR} \rightarrow \frac{1 \times L^2}{2EI} - \frac{M_B L}{EI} = \frac{M_B (2L)}{EI} \rightarrow 3M_B = \frac{L}{2} \rightarrow M_B = \frac{L}{18} \rightarrow M_A = \frac{-4}{9}L^2$$

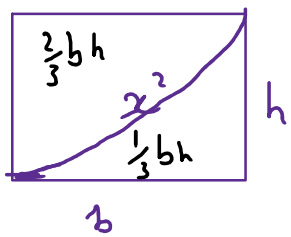
$$\boxed{A^- = \frac{4}{9}L^2}$$

مساحت ناحیه مثبت



$$\theta_{BL} = \theta_{BR} \rightarrow \frac{M_B L}{EI} = \frac{1 (2L)^3}{2EI} - \frac{M_B (2L)}{EI} \rightarrow 3M_B = \frac{4}{3}L^2 \rightarrow M_B = \frac{4}{9}L^2 \rightarrow M_A = M_B$$

$$\boxed{A^+ = \frac{4}{9}L^2}$$



$$A^+ = \frac{1}{3}\left(\frac{2}{3}L\right)(2L) = \frac{4}{9}L^2$$