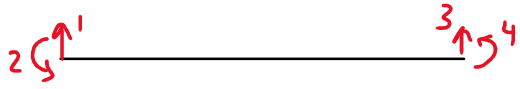
Matrix Analysis 8

Wednesday, July 17, 2024 11:48

تحلیل ماتریسی تیر و قاب ها

① ماتریس سختی عضو در مختصات محلی (Local)

ماتریس سختی عضو خمشی با ۴ درجه آزادی (در مختصات محلی (Local))



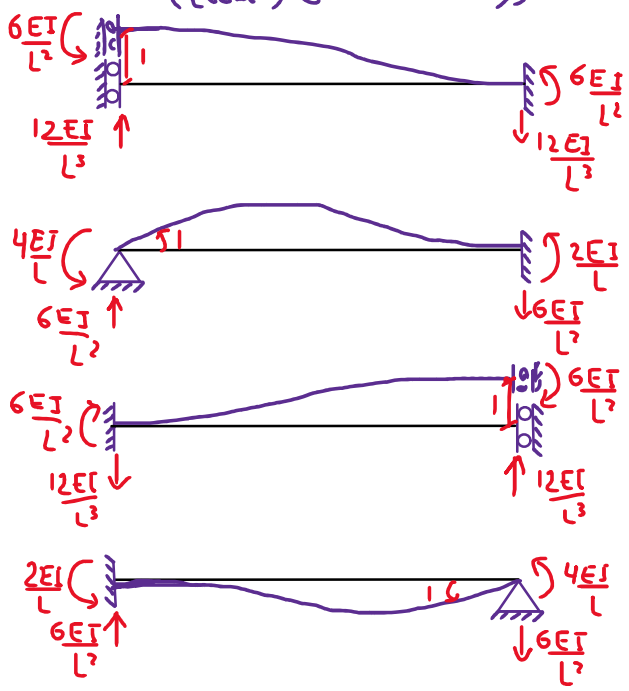
$$k_l = \begin{array}{c} \begin{matrix} ① & ② & ③ & ④ \end{matrix} \\ \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \end{array} \begin{matrix} ① \\ ② \\ ③ \\ ④ \end{matrix}$$

$k_{\delta\delta}$ $\frac{12EI}{L^3}$ 1,3

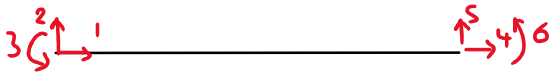
$k_{\delta\theta}$ $\frac{6EI}{L^2}$

$k_{\theta\theta}$ $\frac{4EI}{L}, \frac{2EI}{L}$

$$M_{AB} = \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

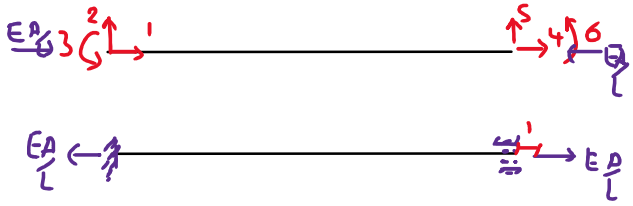


ماتریس سختی عضو خمشی با ۶ درجه آزادی (در مختصات محلی (Local))



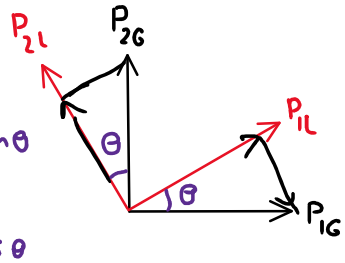
$$k_l = \begin{array}{c} \begin{matrix} ① & ② & ③ & ④ & ⑤ & ⑥ \end{matrix} \\ \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \end{array} \begin{matrix} ① \\ ② \\ ③ \\ ④ \\ ⑤ \\ ⑥ \end{matrix}$$

ماتریس سختی عضو خمشی با ۶ درجه آزادی (در مختصات محلی (Local))



$$k_l = \begin{array}{c} \begin{matrix} 2 & 3 & 5 & 6 \\ ① & ② & ③ & ④ \end{matrix} \\ \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \end{array} \begin{matrix} 1,2 \\ 2,3 \\ 3,5 \\ 4,6 \end{matrix}$$

② ماتریس مبدل یا ماتریس انتقال (R)



$$P_{1L} = P_{1G}\cos\theta + P_{2G}\sin\theta$$

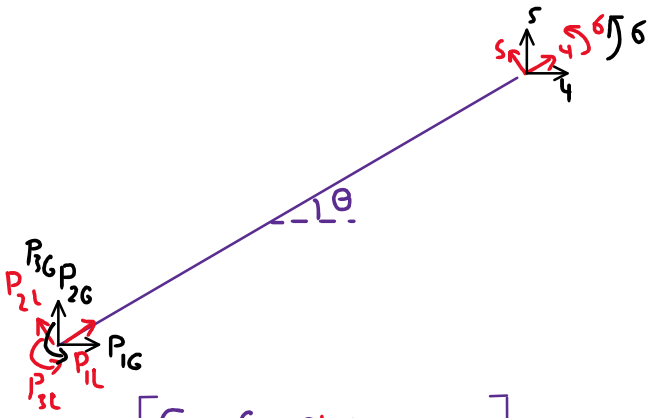
$$P_{2L} = -P_{1G}\sin\theta + P_{2G}\cos\theta$$

$$P_{3L} = P_{3G}$$

$$P_{4L} = P_{4G}\cos\theta + P_{5G}\sin\theta$$

$$P_{5L} = -P_{4G}\sin\theta + P_{5G}\cos\theta$$

$$P_{6L} = P_{6G}$$



$$R = \begin{bmatrix} C & S & 0 & 0 & 0 & 0 \\ -S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C & S & 0 \\ 0 & 0 & 0 & -S & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

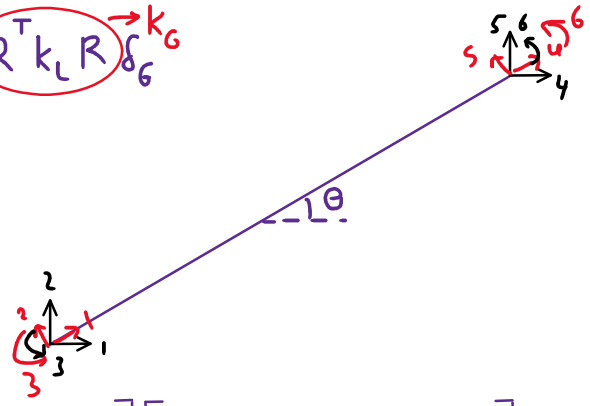
$$P_L = R\,P_G \qquad \delta_L = R\,\delta_G$$

$$R = \begin{bmatrix} R_J & 0 \\ 0 & R_J \end{bmatrix} \qquad R_J = \begin{bmatrix} C & S & 0 \\ -S & C & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

ماتریس سختی عضو در مختصات کلی (Global) ③

$$P_L = k_L\,\delta_L \longrightarrow R\,P_G = k_L\,R\,\delta_G \longrightarrow P_G = \overbrace{R^T k_L R}^{k_G}\,\delta_G$$

$$k_G = R^T k_L R$$



$$k_G = \begin{bmatrix} C & -S & 0 & 0 & 0 & 0 \\ S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C & -S & 0 \\ 0 & 0 & 0 & S & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} C & S & 0 & 0 & 0 & 0 \\ -S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C & S & 0 \\ 0 & 0 & 0 & -S & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$K_G = \left[\begin{array}{ccc|ccc} c^2\frac{EA}{L}+s^2\frac{12EI}{L^3} & cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & -s\frac{6EI}{L^2} & -c^2\frac{EA}{L}-s^2\frac{12EI}{L^3} & -cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & -s\frac{6EI}{L^2} \\ cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & s^2\frac{EA}{L}+c^2\frac{12EI}{L^3} & c\frac{6EI}{L^2} & -cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & -s^2\frac{EA}{L}-c^2\frac{12EI}{L^3} & c\frac{6EI}{L^2} \\ -s\frac{6EI}{L^2} & c\frac{6EI}{L^2} & \frac{4EI}{L} & s\frac{6EI}{L^2} & -c\frac{6EI}{L^2} & \frac{2EI}{L} \\ \hline -c^2\frac{EA}{L}-s^2\frac{12EI}{L^3} & -cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & s\frac{6EI}{L^2} & c^2\frac{EA}{L}+s^2\frac{12EI}{L^3} & cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & s\frac{6EI}{L^2} \\ -cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & -s^2\frac{EA}{L}-c^2\frac{12EI}{L^3} & -c\frac{6EI}{L^2} & cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & s^2\frac{EA}{L}+c^2\frac{12EI}{L^3} & -c\frac{6EI}{L^2} \\ -s\frac{6EI}{L^2} & c\frac{6EI}{L^2} & \frac{2EI}{L} & s\frac{6EI}{L^2} & -c\frac{6EI}{L^2} & \frac{4EI}{L} \end{array}\right]$$