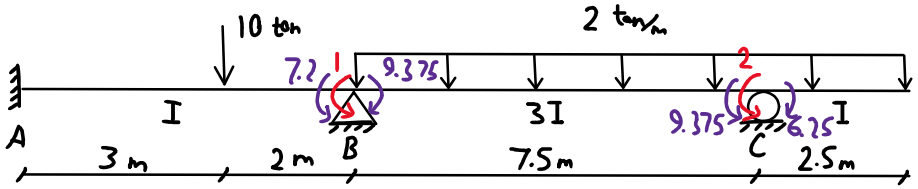
Matrix Analysis 9

Wednesday, July 17, 2024 11:48

تحلیل ماتریسی سازه ها

مثال: تیر شکل زیر را به روش ماتریسی تحلیل کنید.



$$k_{ff}\,\delta_f = P_f \qquad P_L = k_L\,\delta_L + FER$$

$$[k_{ff}]_{AB} = \left[\frac{4EI}{5}\right] = EI\,[0.8] \quad ①$$

* ماتریس سختی k_{ff} اعضا

$$[k_{ff}]_{BC} = \begin{bmatrix} \frac{4(3EI)}{7.5} & \frac{2(3EI)}{7.5} \\ \frac{2(3EI)}{7.5} & \frac{4(3EI)}{7.5} \end{bmatrix} = EI \begin{bmatrix} 1.6 & 0.8 \\ 0.8 & 1.6 \end{bmatrix} \begin{matrix} ① \\ ② \end{matrix}$$

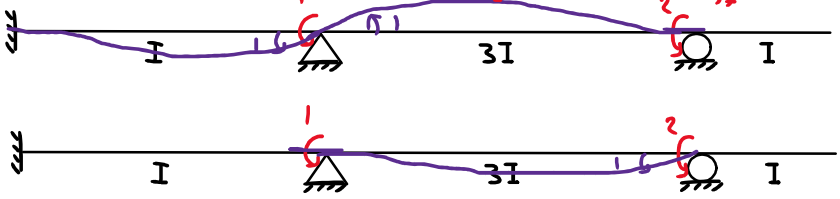
$$k_G = k_L = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{matrix} ① \\ ② \\ ③ \\ ④ \end{matrix}$$

$$\begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix}$$

* ماتریس سختی k_{ff} سازه با سوهم بندی

$$k_{ff} = EI \begin{bmatrix} 2.4 & 0.8 \\ 0.8 & 1.6 \end{bmatrix} \begin{matrix} ① \\ ② \end{matrix}$$

ماتریس k_{ff} به صورت مستقیم



* محاسبه بارگره ای P_f

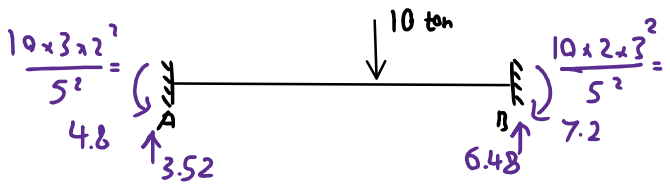
$$P_f = \begin{Bmatrix} 7.2 - 9.375 \\ 9.375 - 6.25 \end{Bmatrix} = \begin{Bmatrix} -2.175 \\ 3.125 \end{Bmatrix} \begin{matrix} ① \\ ② \end{matrix}$$

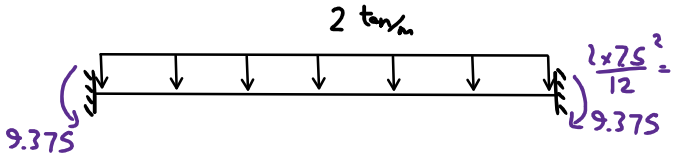
* محاسبه تغییرمکان های گره ای δ_f

$$EI \begin{bmatrix} 2.4 & 0.8 \\ 0.8 & 1.6 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \begin{Bmatrix} -2.175 \\ 3.125 \end{Bmatrix}$$

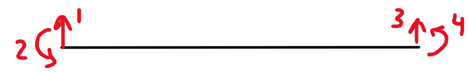
$$\begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \frac{1}{EI} \begin{Bmatrix} -1.87 \\ 2.89 \end{Bmatrix}$$

$$k_{ff} = \begin{bmatrix} \frac{4EI}{5} + \frac{4(3EI)}{7.5} & \frac{2(3EI)}{7.5} \\ \frac{2(3EI)}{7.5} & \frac{4(3EI)}{7.5} \end{bmatrix} \begin{matrix} ① \\ ② \end{matrix} \qquad \begin{matrix} ① & ② \end{matrix}$$



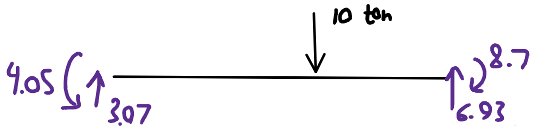


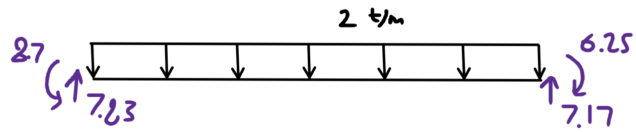
$$(P_L)_{AB} = k_L\delta_L + FER = \begin{bmatrix} & & & \frac{6EI}{5^2} \\ & & & \frac{2EI}{5} \\ & & & -\frac{6EI}{5^2} \\ & & & \frac{4EI}{5} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -\frac{1.87}{EI} \end{Bmatrix} + \begin{Bmatrix} 3.52 \\ 4.8 \\ 6.48 \\ -7.2 \end{Bmatrix}$$



$$k_L = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

$$(P_L)_{AB} = \begin{Bmatrix} 3.07 \\ 4.05 \\ 6.93 \\ -8.7 \end{Bmatrix}$$



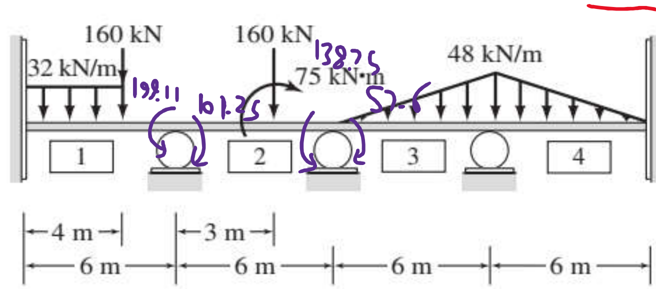


$$(P_L)_{BC} = k_L\delta_L + FER = \begin{bmatrix} & \frac{6(3EI)}{7.5^2} & & \frac{6(3EI)}{7.5^2} \\ & \frac{4(3EI)}{7.5} & & \frac{2(3EI)}{7.5} \\ & -\frac{6(3EI)}{7.5^2} & & -\frac{6(3EI)}{7.5^2} \\ & \frac{4(3EI)}{7.5} & & \frac{4(3EI)}{7.5} \end{bmatrix} \begin{Bmatrix} 0 \\ -\frac{1.87}{EI} \\ 0 \\ \frac{2.89}{EI} \end{Bmatrix} + \begin{Bmatrix} 7.5 \\ 9.375 \\ 7.5 \\ -9.375 \end{Bmatrix} = \begin{Bmatrix} 7.83 \\ 8.7 \\ 7.17 \\ -6.25 \end{Bmatrix}$$

$$(P_L)_{BC} = \begin{Bmatrix} 7.83 \\ 8.7 \\ 7.17 \\ -6.25 \end{Bmatrix}$$

تیرها با بارگذاری های متنوع

$$P_f = \begin{Bmatrix} 199.11 - 101.25 \\ 138.75 - 576 \\ . \end{Bmatrix}$$



$E = 200$ GPa
$I = 120(10^6)$ mm^4

از جدول انتهای کتاب کاسیمالی (در سایت daneshesaze.com)

Member 1: (Loading types 1 and 3)

$$FS_b = \frac{160(2)^2(14)}{(6)^3} + \frac{32(6)}{2}\left[1 - \frac{(2)^3(10)}{(6)^4}\right] = 131.56 \text{ kN}$$

$$FM_b = \frac{160(4)(2)^2}{(6)^2} + \frac{32(6)^2}{12}\left[1 - \frac{(2)^3(18)}{(6)^4}\right] = 156.44 \text{ kN}\cdot\text{m}$$

$$FS_e = \frac{160(4)^2(10)}{(6)^3} + \frac{32(6)}{2}\left[1 - \frac{2(392)}{(6)^4}\right] = 156.44 \text{ kN}$$

$$FM_e = -\frac{160(4)^2(2)}{(6)^2} - \frac{32(6)^2}{12}\left[1 - \frac{(2)^2(132)}{(6)^4}\right] = -199.11 \text{ kN}\cdot\text{m}$$

Member 2: (Loading types 1 and 2)

$$FS_b = 80 - \frac{6(75)(3)^2}{(6)^3} = 61.25 \text{ kN}$$

$$FM_b = \frac{160(3)^3}{(6)^2} + \frac{75(3)(-3)}{(6)^2} = 101.25 \text{ kN}\cdot\text{m}$$

$$FS_e = 80 + \frac{6(75)(3)^2}{(6)^3} = 98.75 \text{ kN}$$

$$FM_e = -\frac{160(3)^3}{(6)^2} + \frac{75(3)(-3)}{(6)^2} = -138.75 \text{ kN}\cdot\text{m}$$

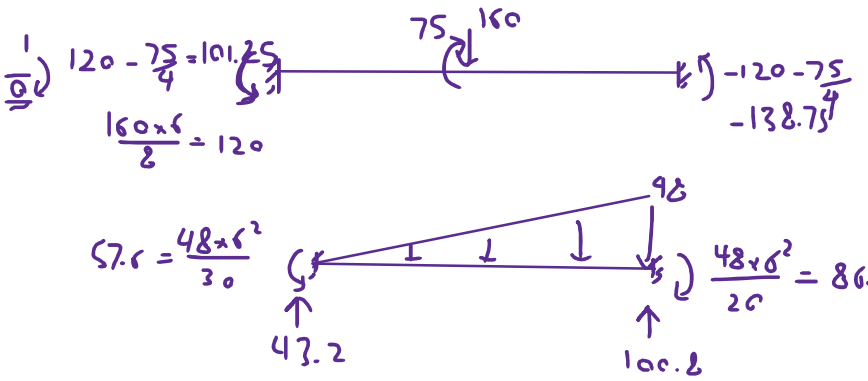
Member 3: (Loading type 4)

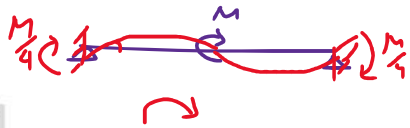
$$FS_b = \frac{48(6)^3(3)(6)}{20(6)^3} = 43.2\text{kN}$$

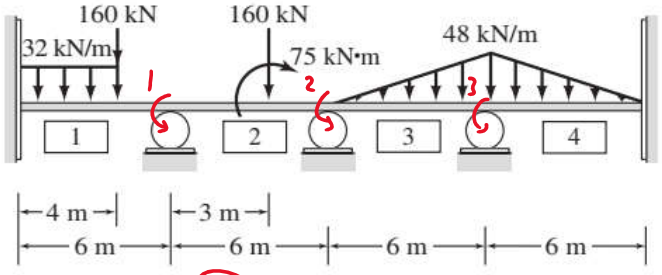
$$FM_b = \frac{48(6)^3(2)(6)}{60(6)^2} = 57.6 \text{ kN}\cdot\text{m}$$

$$FS_e = \left(\frac{48}{2}\right)(6) - 43.2 = 100.8 \text{ kN}$$

$$FM_e = (-48)(6) + (43.2)(6) - 57.6 = -86.4 \text{ kN}\cdot\text{m}$$







$E = 200$ GPa

$I = 120(10^6)$ mm^4

$$k_{ff} = \begin{bmatrix} \frac{4EI_1}{L_1} + \frac{4EI_2}{L_2} & \frac{2EI_2}{L_2} & 0 \\ \frac{2EI_2}{L_2} & \frac{4EI_2}{L_2} + \frac{4EI_3}{L_3} & \frac{2EI_3}{L_3} \\ 0 & \frac{2EI_3}{L_3} & \frac{4EI_3}{L_3} + \frac{4EI_4}{L_4} \end{bmatrix} \quad P_f = \begin{Bmatrix} \\ \\ \end{Bmatrix}$$