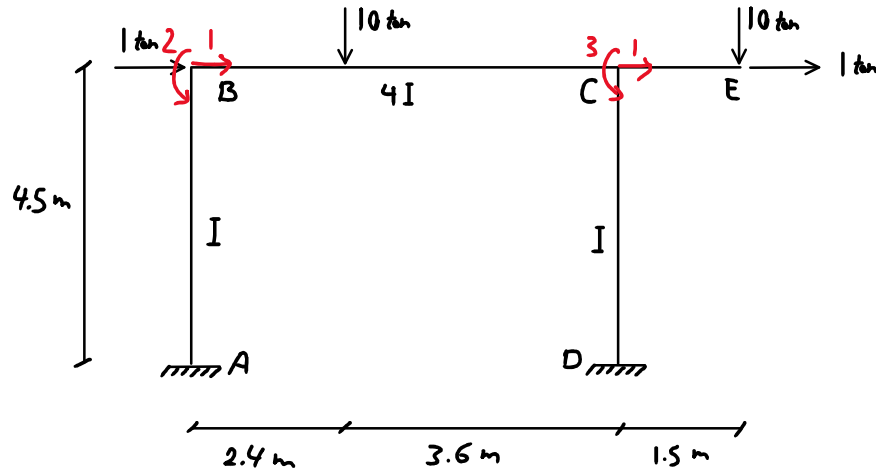


مثال: قاب شکل زیر را با روش ماتريسي تحليل کنید. (از تغيير طول محورهاى اعضا صرف نظر کنید).



① $k_{ff} \delta_f = P_f$

② $P_i = k_{ij} \delta_j + F_{ER}$

① تشکیل ماتريسي گسسته k_{ff}

$$[k_{ff}]_{AB} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} = EI \begin{bmatrix} \frac{12}{4.5^3} & \frac{6}{4.5^2} \\ \frac{6}{4.5^2} & \frac{4}{4.5} \end{bmatrix} = EI \begin{bmatrix} 0.1317 & 0.2963 \\ 0.2963 & 0.8889 \end{bmatrix}$$

$$[k_{ff}]_{CD} = EI \begin{bmatrix} 0.1317 & 0.2963 \\ 0.2963 & 0.8889 \end{bmatrix}$$

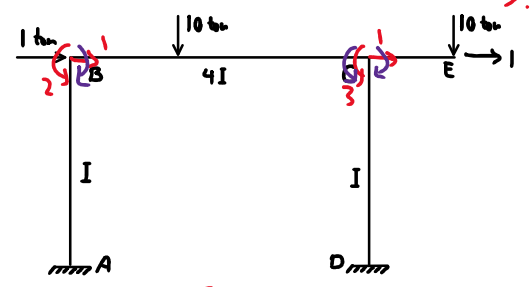
$$[k_{ff}]_{BC} = \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix} = \begin{bmatrix} \frac{4(4EI)}{6} & \frac{2(4EI)}{6} \\ \frac{2(4EI)}{6} & \frac{4(4EI)}{6} \end{bmatrix} = EI \begin{bmatrix} 2.6667 & 1.3333 \\ 1.3333 & 2.6667 \end{bmatrix}$$

سره هم بند

$$k_{ff} = EI \begin{bmatrix} 0.2634 & 0.2963 & 0.2963 \\ 0.2963 & 3.5556 & 1.3333 \\ 0.2963 & 1.3333 & 3.5556 \end{bmatrix}$$

بردار بازنه‌ها را برعکس

$$P_f = \begin{Bmatrix} 1+1 \\ -\frac{10 \times 2.4 \times 3.6^2}{6^2} \\ \frac{10 \times 3.6 \times 2.4^2}{6^2} - 10 \times 1.5 \end{Bmatrix} = \begin{Bmatrix} 2 \\ -8.64 \\ -9.24 \end{Bmatrix}$$



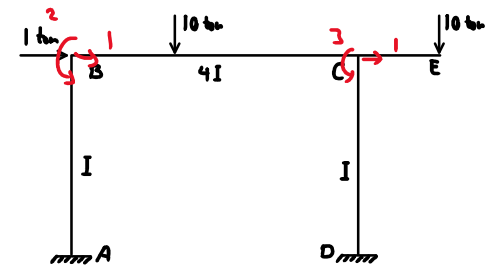
محاسبه محمولات تغییر مکانی (دره)

$$EI \begin{bmatrix} 0.2963 & 0.2963 & 0.2963 \\ 0.2967 & 3.5556 & 1.3333 \\ 0.2963 & 1.3333 & 3.5556 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{Bmatrix} = \begin{Bmatrix} 2 \\ -8.64 \\ -9.24 \end{Bmatrix} \rightarrow \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{Bmatrix} = \frac{1}{EI} \begin{Bmatrix} 13.555 \\ -2.515 \\ -2.785 \end{Bmatrix}$$

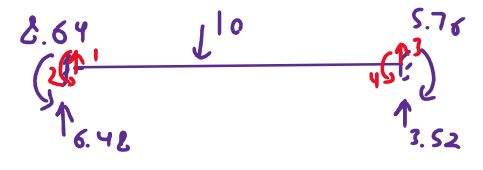
محاسبه نیروها

$$P_L = k_L \delta_L + F_{ER}$$

$$(P_L)_{AB} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ -\delta_1 \\ \delta_2 \end{Bmatrix} + 0 = \begin{Bmatrix} 1.0398 \\ 2.8986 \\ -1.0398 \\ 1.7807 \end{Bmatrix}$$

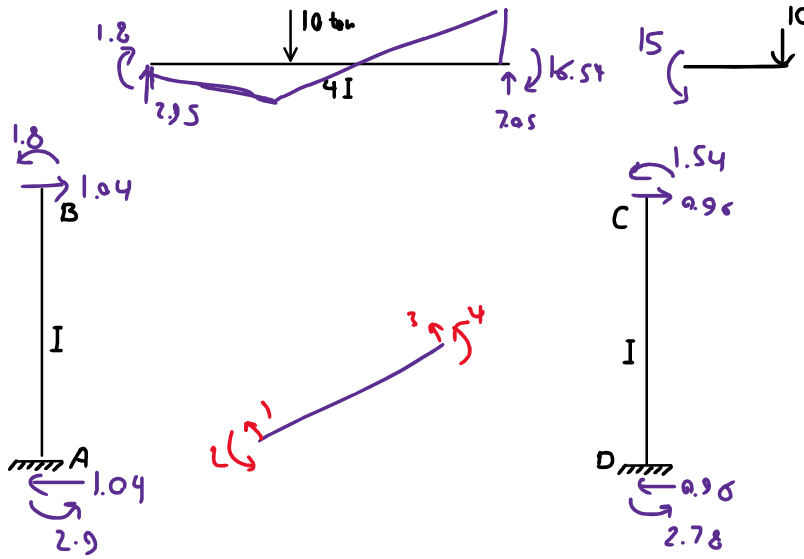


$$(P_L)_{CD} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ -\delta_1 \\ \delta_3 \end{Bmatrix} + 0 = \begin{Bmatrix} 0.9598 \\ 2.7786 \\ -0.9598 \\ 1.5407 \end{Bmatrix}$$



$$(P_L)_{BC} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{Bmatrix} 0 \\ \delta_2 \\ 0 \\ \delta_3 \end{Bmatrix} + \begin{Bmatrix} 6.48 \\ 8.64 \\ 3.52 \\ -5.76 \end{Bmatrix} = \begin{Bmatrix} 2.9464 \\ -1.7807 \\ 7.0536 \\ -16.5407 \end{Bmatrix}$$

$$P_{AB} = \begin{Bmatrix} 1.0398 \\ 2.8986 \\ -1.0398 \\ 1.7807 \end{Bmatrix}, P_{CD} = \begin{Bmatrix} 0.9596 \\ 2.7786 \\ -0.9598 \\ 1.5407 \end{Bmatrix}, P_{BC} = \begin{Bmatrix} 2.9464 \\ -1.7807 \\ 7.0536 \\ -16.5407 \end{Bmatrix}$$



نوشته ماتریس سختی k_{ff} به صورت مستقیم

$$k_{ff} = EI \begin{bmatrix} \textcircled{1} & \textcircled{2} & \textcircled{3} \\ \frac{2 \times 12EI}{4.5^3} & \frac{6EI}{4.5^2} & \frac{6EI}{4.5^2} \\ \frac{6EI}{4.5^2} & \frac{4EI + 4(4EI)}{4.5} & \frac{2(4EI)}{6} \\ \frac{6EI}{4.5^2} & \frac{2(4EI)}{6} & \frac{4EI + 4(4EI)}{4.5} \end{bmatrix} \begin{matrix} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \end{matrix}$$

$$k_{ff} = EI \begin{bmatrix} \textcircled{1} & \textcircled{2} & \textcircled{3} \\ 0.2634 & 0.2963 & 0.2963 \\ 0.2967 & 3.5556 & 1.3333 \\ 0.2963 & 1.3333 & 3.5556 \end{bmatrix} \begin{matrix} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \end{matrix}$$

