Matrix Analysis 21

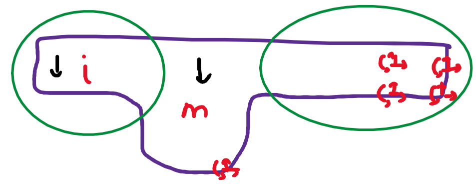
Sunday, September 15, 2024 13:12

تراکم استاتیکی ماتریس سختی (Static Condensation)

ماتریس سختی سازه را می‌توان با حذف یک سری از درجات آزادی و لحاظ کردن اثر آنها در سختی درجات آزادی دیگر متراکم نمود.

δ_m : درجات آزادی اصلی

δ_i : سایر درجات آزادی



$$\left[\begin{array}{c:c} k_{mm} & k_{mi} \\ \hdashline k_{im} & k_{ii} \end{array}\right]\left\{\begin{array}{c} \delta_m \\ \hdashline \delta_i \end{array}\right\}=\left\{\begin{array}{c} P_m \\ \hdashline P_i \end{array}\right\} \rightarrow \begin{cases} (1)\quad k_{mm}\delta_m + k_{mi}\delta_i = P_m \\ (2)\quad k_{im}\delta_m + k_{ii}\delta_i = P_i \rightarrow \delta_i = k_{ii}^{-1}(P_i - k_{im}\delta_m) \end{cases}$$

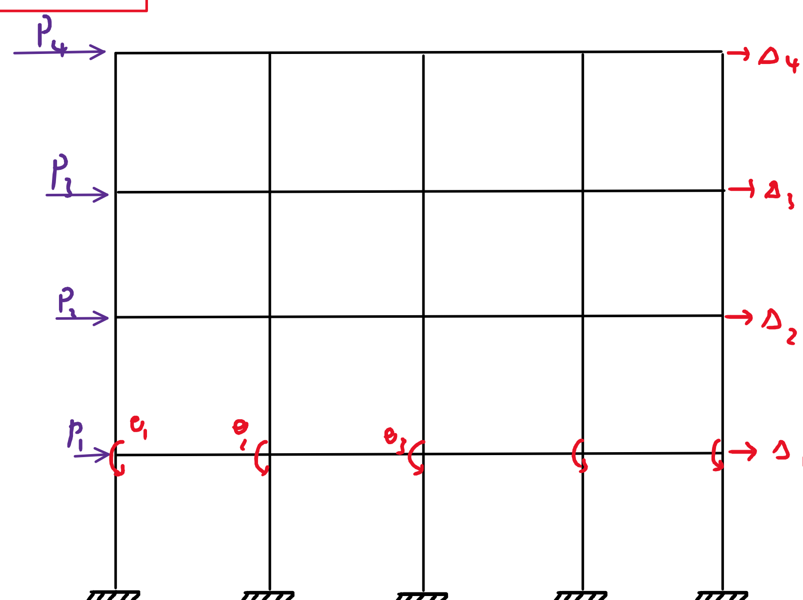
جایگذاری δ_i در معادله (1):

$$k_{mm}\delta_m + k_{mi}k_{ii}^{-1}(P_i - k_{im}\delta_m) = P_m$$

$$\boxed{(\underbrace{k_{mm} - k_{mi}k_{ii}^{-1}k_{im}}_{k})\,\delta_m = \underbrace{P_m - k_{mi}k_{ii}^{-1}P_i}_{P}}$$

با صرف نظر از تغییر شکل محوری اعضا

$$\delta_f : 24 \begin{cases} 4\,\Delta \\ 20\,\theta \end{cases}$$



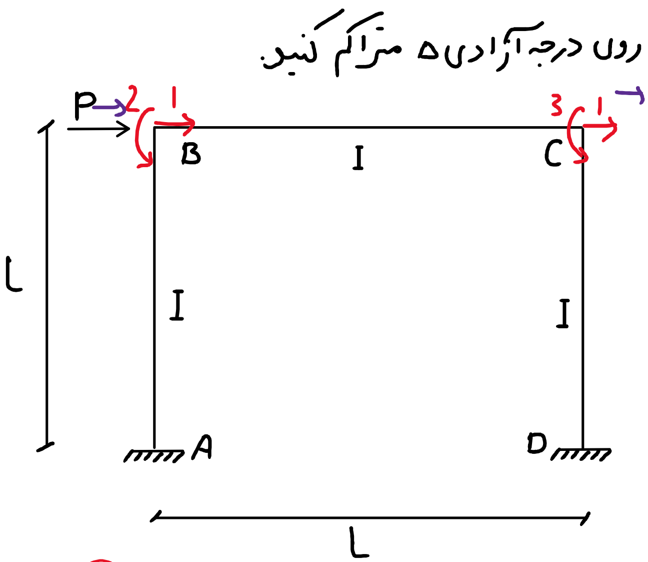
$$\begin{array}{c} (1) \\ (2) \end{array}\left[\begin{array}{c:c} \underset{4\times4}{k_{\Delta\Delta}} & \underset{4\times20}{k_{\Delta\theta}} \\ \hdashline k_{\theta\Delta} & k_{\theta\theta} \end{array}\right]\left\{\begin{array}{c} \underset{4\times1}{\Delta} \\ \hdashline \theta \end{array}\right\}=\left\{\begin{array}{c} \underset{4\times1}{P} \\ \hdashline 0 \end{array}\right\}$$

$$\begin{array}{cccc} 20\times4 & 20\times20 & 20\times1 & 20\times1 \end{array}$$

$$(2)\quad k_{\theta\Delta}\Delta + k_{\theta\theta}\theta = 0 \rightarrow \boxed{\theta = -k_{\theta\theta}^{-1}k_{\theta\Delta}\Delta}$$

$$(1)\quad k_{\Delta\Delta}\Delta + k_{\Delta\theta}(-k_{\theta\theta}^{-1}k_{\theta\Delta})\Delta = P \rightarrow \boxed{(\overbrace{\underset{4\times4}{k_{\Delta\Delta}} - \underset{4\times20}{k_{\Delta\theta}}\underset{20\times20}{k_{\theta\theta}^{-1}}\underset{20\times4}{k_{\theta\Delta}}}^{k_{4\times4}})\,\Delta = P}$$

مثال : ماتریس سختی قاب شکل زیر را روی درجه آزادی Δ متراکم کنید.



$$
\begin{array}{cc}
 & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\
k_{ff}=EI & \left[\begin{array}{c|cc} \frac{24EI}{L^3} & \frac{6EI}{L^2} & \frac{6EI}{L^2} \\ \hline \frac{6EI}{L^2} & \frac{8EI}{L} & \frac{2EI}{L} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & \frac{8EI}{L} \end{array}\right]
\end{array}
\begin{array}{c} 1 \\ 2 \\ 3 \end{array}
$$

$k_{\Delta\Delta}$, $k_{\Delta\theta}$, $k_{\theta\Delta}$, $k_{\theta\theta}$

$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow A^{-1}=\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$k_{ff,Con}=k_{\Delta\Delta}-k_{\Delta\theta}k_{\theta\theta}^{-1}k_{\theta\Delta}=\frac{24EI}{L^3}-\overbrace{\left\{\frac{6EI}{L^2}\quad\frac{6EI}{L^2}\right\}}^{k_{\Delta\theta}}\overbrace{\frac{1}{\left(\frac{EI}{L}\right)^2(8^2-2^2)}\begin{bmatrix} \frac{8EI}{L} & -\frac{2EI}{L} \\ -\frac{2EI}{L} & \frac{8EI}{L} \end{bmatrix}}^{k_{\theta\theta}^{-1}}\overbrace{\begin{Bmatrix} \frac{6EI}{L^2} \\ \frac{6EI}{L^2} \end{Bmatrix}}^{k_{\theta\Delta}}$$

$$=\frac{24EI}{L^3}-\overbrace{\left(\frac{6EI}{\cancel{L^2}}\right)\left(\frac{1}{\cancel{60}_{10}\left(\frac{EI}{\cancel{L}}\right)^{\cancel{2}}}\frac{\cancel{EI}}{\cancel{L}}\right)\left(\frac{6EI}{L^2}\right)}^{0.6\frac{EI}{L^3}}\{1\quad 1\}\begin{bmatrix} 8 & -2 \\ -2 & 8 \end{bmatrix}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \quad \begin{Bmatrix} 6 \\ 6 \end{Bmatrix}$$

$$k_{ff,Con}=\frac{24EI}{L^3}-0.6\frac{EI}{L^3}\{1\quad 1\}\begin{Bmatrix} 6 \\ 6 \end{Bmatrix}=(24-0.6\times 12)\frac{EI}{L^3}=\boxed{16.8\frac{EI}{L^3}} \qquad \frac{168}{10}=\frac{84}{5}$$

$$\boxed{P=k\Delta \;\Rightarrow\; P=\frac{84}{5}\frac{EI}{L^3}\Delta}$$

$$\theta=-k_{\theta\theta}^{-1}k_{\theta\Delta}\Delta=-\frac{1}{60\left(\frac{EI}{L}\right)^2}\begin{bmatrix} \frac{8EI}{L} & -\frac{2EI}{L} \\ -\frac{2EI}{L} & \frac{8EI}{L} \end{bmatrix}\begin{Bmatrix} \frac{6EI}{L^2} \\ \frac{6EI}{L^2} \end{Bmatrix}\Delta$$

$$\theta = \frac{-1}{\cancel{60}^{\,10}\cancel{\left(\frac{EI}{L}\right)^2}} \cancel{\left(\frac{EI}{L}\right)} \left(\frac{6EI}{L^2}\right) \begin{bmatrix} 8 & -2 \\ -2 & 8 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \Delta = -\frac{6}{10} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \frac{\Delta}{L}$$

$$\boxed{\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} -\frac{6}{10}\frac{\Delta}{L} \\ -\frac{6}{10}\frac{\Delta}{L} \end{Bmatrix}}$$

$$P_{L,AB} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & \frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ -\Delta \\ -\frac{6}{10}\frac{\Delta}{L} \end{Bmatrix} + \cancel{FER}^{\,0} = \begin{Bmatrix} 8.4\frac{EI}{L^3}\Delta \\ 4.8\frac{EI}{L^2}\Delta \\ -8.4\frac{EI}{L^3}\Delta \\ 3.6\frac{EI}{L^2}\Delta \end{Bmatrix} = \begin{Bmatrix} P/2 \\ \frac{2}{7}PL \\ -P/2 \\ \frac{3}{14}PL \end{Bmatrix}$$

$\Delta = \frac{5}{84}\frac{PL^3}{EI}$