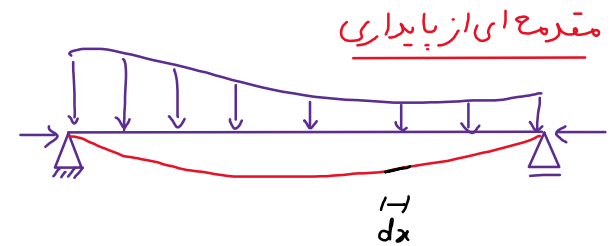


کمانش پیچشی ستون ها



$$\sum F_y = 0$$

$$-w dx - (V + dV) = 0 \rightarrow \frac{dV}{dx} = -w$$

انتگرال

$$\left\{ \begin{array}{l} V = \frac{dM}{dx} \\ w = \frac{dV}{dx} \end{array} \right.$$

$$\sum M_o = 0$$

$$M + V dx - P dy - (M + dM) - w \frac{dx^2}{2} = 0$$

$$V = \frac{dM}{dx} + P y' \rightarrow w = \frac{d^2 M}{dx^2} + P y'' = 0$$

معادله دیفرانسیل کمانش پیچشی ستون ها

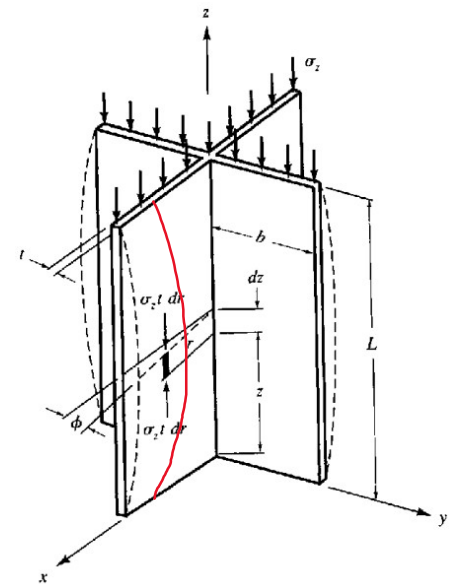
$$T = T_s + T_w$$

$$t_u = \frac{dT}{dz} = 0$$

بارگذاری پیچشی در طول عضو نداریم.

$$\frac{dT_s}{dz} + \frac{dT_w}{dz} = 0$$

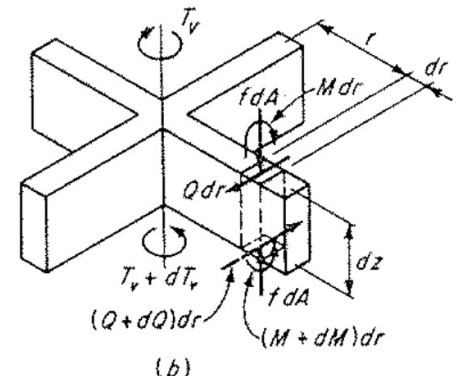
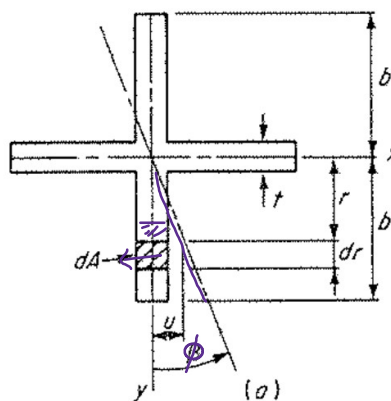
$$T_s = GJ \frac{d\phi}{dz} \rightarrow \frac{dT}{dz} = GJ \frac{d^2 \phi}{dz^2}$$



$$T_w = \int r Q dr$$

$$\frac{dT_w}{dz} = \int r \frac{dQ}{dz} dr$$

Q: بیش در واحد عرض
M: کمتر در واحد عرض



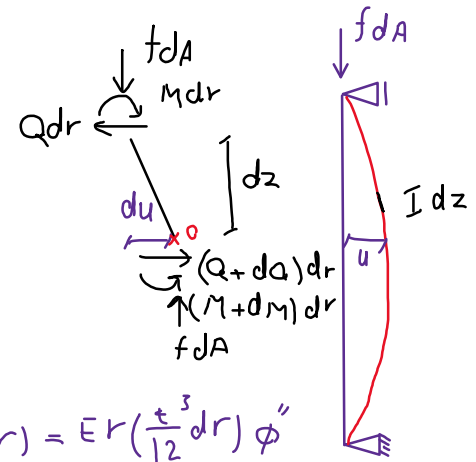
$$\sum M_o = 0$$

$$\frac{1}{dz} Q dr dz - M dr + f dA du + (M + dM) dr = 0$$

$$\frac{d}{dz} Q dr + f dA \frac{du}{dz} + \frac{dM}{dz} dr = 0$$

$$\frac{dQ}{dz} dr + f dA \frac{d^2 u}{dz^2} + \frac{d^2 M}{dz^2} dr = 0$$

$$\begin{cases} u = r\phi \rightarrow u'' = r\phi'' \\ M dr = EI u'' = E \left(\frac{t^3}{12} dr \right) = E r \left(\frac{t^3}{12} dr \right) \phi'' \end{cases}$$



$$\frac{dQ}{dz} dr + f dA r \frac{d^2 \phi}{dz^2} + E r \left(\frac{t^3}{12} dr \right) \frac{d^4 \phi}{dz^4} = 0$$

$$\frac{dT_w}{dz} = \int r \frac{dQ}{dz} dr = -f \int r^2 dA \frac{d^2 \phi}{dz^2} - E \frac{t^3}{12} \int r^2 dr \frac{d^4 \phi}{dz^4}$$

$$\frac{dT_w}{dz} = -f I_p \frac{d^2 \phi}{dz^2} - E \left(4 \times \frac{b^3 t^3}{36} \right) \frac{d^4 \phi}{dz^4}$$

$$\frac{dT_w}{dz} = -f I_p \frac{d^2 \phi}{dz^2} - E C_w \frac{d^4 \phi}{dz^4}$$

$$\frac{dT_s}{dz} + \frac{dT_w}{dz} = 0$$

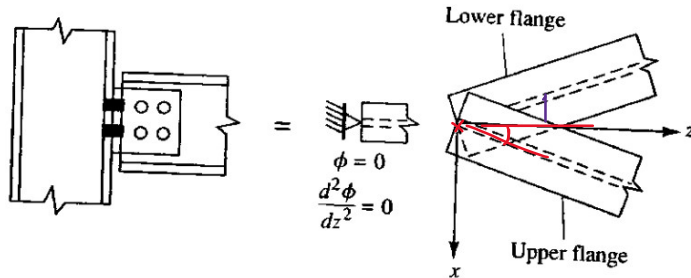
$$GJ \frac{d^2 \phi}{dz^2} - f I_p \frac{d^2 \phi}{dz^2} - E C_w \frac{d^4 \phi}{dz^4} = 0$$

$$\frac{d^4 \phi}{dz^4} + \left(\frac{f I_p - GJ}{E C_w} \right) \frac{d^2 \phi}{dz^2} = 0$$

$$u = r\phi$$

$$u = 0 \rightarrow \phi = 0$$

$$M = EI u'' = 0 \rightarrow \phi'' = 0$$



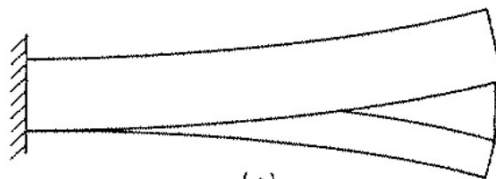
شرایط مرزی

① مفصلی

$$\begin{cases} \phi = 0 \\ \phi'' = 0 \end{cases}$$

③ گیردار

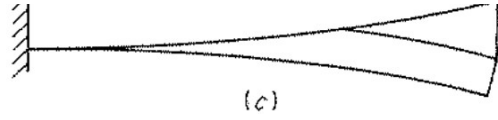
$$\begin{cases} u = 0 \rightarrow \phi = 0 \\ u' = 0 \rightarrow \phi' = 0 \end{cases}$$



② آزاد

$$\begin{aligned} M = EI u'' = 0 &\rightarrow \phi'' = 0 \\ T = 0 &\rightarrow (f I_p - GJ) \phi' + E C_w \phi''' = 0 \\ \phi''' + \lambda^2 \phi' &= 0 \end{aligned}$$

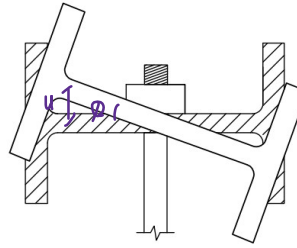
$$\{ u' = 0 \rightarrow \phi = 0$$



$$T = 0 \rightarrow (f I_p - GJ) \phi' + E C_w \phi'' = 0$$

$$\phi''' + \lambda^2 \phi' = 0$$

$$\begin{cases} u = r \phi \\ u' = r \phi' \\ u'' = r \phi'' \end{cases}$$



جلد پراپی لید (آزاد)

* کمائن، همیشه آزاد

* کمائن، فن حول محور چین - منقل

نش کمائن پیچش (F_e) برای ستون دوسرگیردار

λ^2

$$\frac{d^4 \phi}{dz^4} + \left(\frac{f I_p - GJ}{E C_w} \right) \frac{d^2 \phi}{dz^2} = 0$$

$$\phi(z) = A_1 \sin \lambda z + A_2 \cos \lambda z + A_3 z + A_4$$

$$\phi(0) = 0 \rightarrow A_2 + A_4 = 0 \rightarrow A_4 = -A_2$$

$$\phi'(0) = 0 \rightarrow A_1 + A_3 = 0 \rightarrow A_3 = -A_1$$

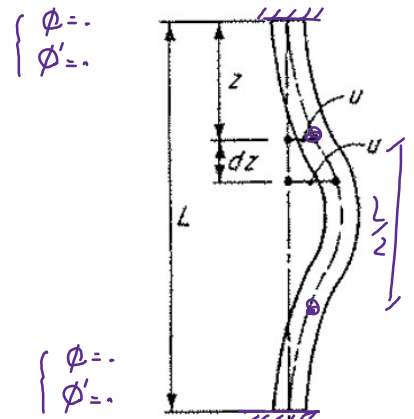
$$\phi(z) = A_1 (\sin \lambda z - \lambda z) + A_2 (\cos \lambda z - 1)$$

$$\phi(L) = 0 \rightarrow A_1 (\sin \lambda L - \lambda L) + A_2 (\cos \lambda L - 1) = 0$$

$$\phi'(L) = 0 \rightarrow A_1 \lambda (\cos \lambda L - 1) + A_2 \lambda (-\sin \lambda L) = 0$$

دو معادله
دو مجهول

$$\rightarrow A_1 = A_2 = 0$$



$$\begin{bmatrix} (\sin \lambda L - \lambda L) & \cos \lambda L - 1 \\ (\cos \lambda L - 1) & -\sin \lambda L \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \rightarrow \begin{vmatrix} (\sin \lambda L - \lambda L) & \cos \lambda L - 1 \\ (\cos \lambda L - 1) & -\sin \lambda L \end{vmatrix} = 0$$

$$+ \sin \lambda L (\sin \lambda L - \lambda L) + (\cos \lambda L - 1)^2 = 0$$

$$\sin^2 \lambda L - \lambda L \sin \lambda L + \cos^2 \lambda L + 1 - 2 \cos \lambda L = 0 \rightarrow \lambda L \sin \lambda L + 2 \cos \lambda L = 2 \rightarrow \lambda L = 2\pi$$

$$\frac{f I_p - GJ}{E C_w} = \left(\frac{\pi}{L} \right)^2$$

$$F_e = \left(\frac{\pi^2}{(0.5L)^2} E C_w + GJ \right) \frac{1}{I_p}$$

دوسرگیردار

$$F_e = \left(\frac{\pi^2}{L^2} E C_w + GJ \right) \frac{1}{I_p}$$

حلبه قبل: دوسرمنقل

$$F_e = \left(\frac{\pi^2 E C_w}{L_{cz}^2} + GJ \right) \frac{1}{I_x + I_y}$$

دجات کلی طول مؤثر $L_{cz} = kL$