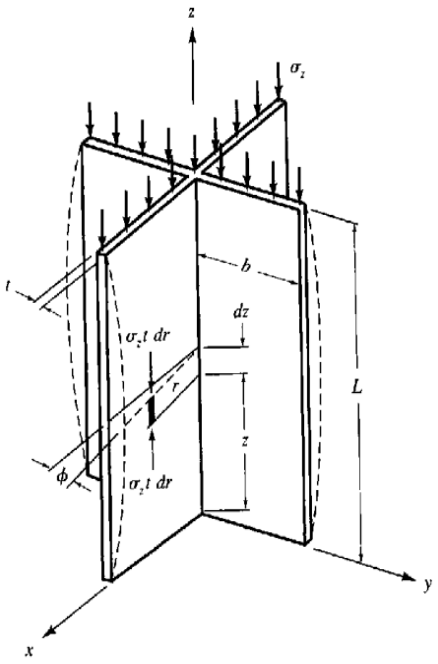
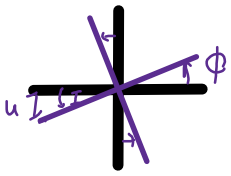
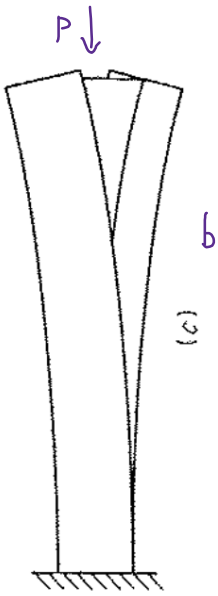
Compression Member 16

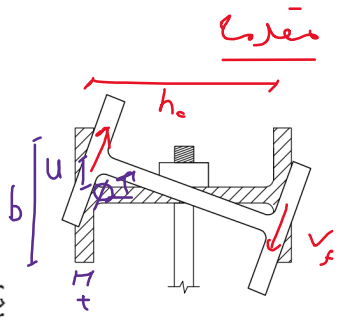
Thursday, July 24, 2025 18:18

کمانش پیچشی ستون‌ها









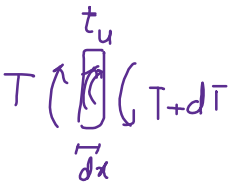
$u = h_o/2\ \phi$

$u''' = \frac{h_o}{2}\phi'''$

$I_f = \frac{1}{12} t b^3$

$M_f = EI_f u'' \rightarrow V_f = EI_f u''' \rightarrow \begin{cases} *T_w = V_f h_o = EI_f u''' h_o = EI_f \frac{h_o}{2}\phi''' h_o = E\left(\frac{I_f h_o^2}{2}\right)\phi''' \\ *T_s = GJ\phi' \end{cases}$ (C_w)

$T = T_s + T_w = GJ\phi' - EC_w\phi'''$



$t_u = \frac{dT}{dz} = GJ\frac{d^2\phi}{dz^2} - EC_w\frac{d^4\phi}{dz^4}$

$dT = t_u dx$

بارگسترده معادل $dw = \sigma\, dA\, u'' = \sigma\, dA\, r\phi''$

بارگسترده پیچشی معادل $t_u = \int r\, dw = \int r^2 \sigma\, dA\, \phi'' = \sigma \underbrace{\int r^2 dA}_{I_p}\phi'' \rightarrow t_u = \sigma I_p \phi''$

$\sigma I_p \phi'' = GJ\phi'' - EC_w\phi^{(4)}$

$EC_w\frac{d^4\phi}{dz^4} + (\sigma I_p - GJ)\frac{d^2\phi}{dz^2} = 0 \rightarrow \frac{d^4\phi}{dz^4} + \left(\frac{\sigma I_p - GJ}{EC_w}\right)\frac{d^2\phi}{dz^2} = 0$ (λ^2)

$M = -Py$

$EIy'' = -Py$

$w = \frac{d^2M}{dx^2} = -py''$ ← بارگسترده معادل

اعمال شرایط مرزی دوسرمفصل $\rightarrow A\sin\lambda L = 0 \rightarrow \lambda L = \pi \rightarrow \lambda^2 = \frac{\pi^2}{L^2}$

$\frac{\sigma I_p - GJ}{EC_w} = \frac{\pi^2}{L^2} \rightarrow \boxed{F_e = \left(\frac{\pi^2}{L^2}EC_w + GJ\right)\frac{1}{I_p}}$ ✓

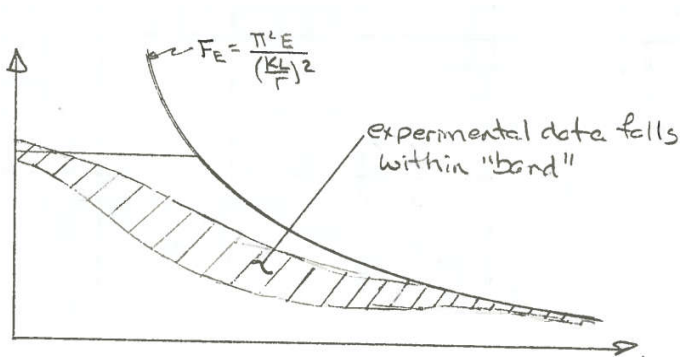
$$\frac{\pi^2 E}{\left(\frac{L}{r_z}\right)^2} = \left(\frac{\pi^2}{L^2} E C_w + GJ\right)\frac{1}{I_p} \rightarrow \frac{\pi^2 E}{L^2} r_z^2 = \frac{\pi^2 E}{L^2}\left(\frac{C_w}{I_p} + \frac{GL^2}{\pi^2 E}\frac{J}{I_p}\right) \rightarrow \boxed{r_z = \sqrt{\frac{C_w}{I_p} + 0.039\frac{JL^2}{I_p}}}$$

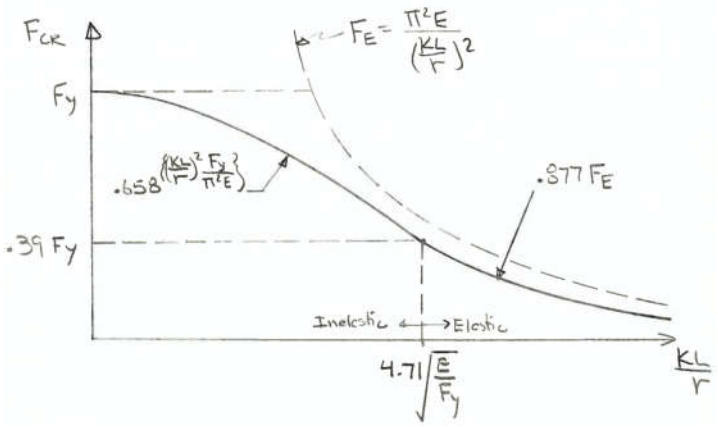
$G = \frac{E}{2(1+\upsilon)} \xrightarrow{\upsilon = 0.3} \frac{E}{G} = 2.6$

* یعنی برای محاسبه تنش کمانش پیچشی توان از رابطه $\frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$ استفاده کرد که در آن r از رابطه فوق به دست می‌آید.

کمانش غیرالاستیک و تنش اسمی (F_n)

اثر تنش پسماند و ناشی قبلی ← جلسه ۲۶





ضوابط آیین نامه‌ای

E4. TORSIONAL AND FLEXURAL-TORSIONAL BUCKLING OF SINGLE ANGLES AND MEMBERS WITHOUT SLENDER ELEMENTS

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This section applies to singly symmetric and unsymmetric members, certain doubly symmetric members, such as cruciform or built-up members, and doubly symmetric members when the torsional unbraced length exceeds the lateral unbraced length, all without slender elements. These provisions also apply to single angles with $b/t > 0.71\sqrt{E/F_y}$, where b is the width of the longest leg and t is the thickness.

The nominal compressive strength, P_n, shall be determined based on the limit states of torsional and flexural-torsional buckling:

$$P_n = F_n A_g \qquad \text{(E4-1)}$$

The nominal stress, F_n, shall be determined according to Equation E3-2 or E3-3, using the torsional or flexural-torsional elastic buckling stress, F_e, determined as follows:

The nominal stress, F_n, is determined as follows:

(a) When $\dfrac{L_c}{r} \le 4.71\sqrt{\dfrac{E}{F_y}}$ (or $\dfrac{F_y}{F_e} \le 2.25$)

$$F_n = \left(0.658^{\frac{F_y}{F_e}}\right) F_y \qquad \text{(E3-2)}$$

AISC 360-22
P40
مبحث دهم ۱۴۰۱
ص ۶۹

(b) When $\dfrac{L_c}{r} > 4.71\sqrt{\dfrac{E}{F_y}}$ (or $\dfrac{F_y}{F_e} > 2.25$)

$$F_n = 0.877 F_e \qquad \text{(E3-3)}$$

(a) For doubly symmetric members twisting about the shear center

$$F_e = \left(\frac{\pi^2 E C_w}{L_{cz}^2} + GJ\right)\frac{1}{I_x + I_y} \qquad \text{(E4-2)}$$

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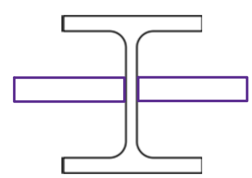
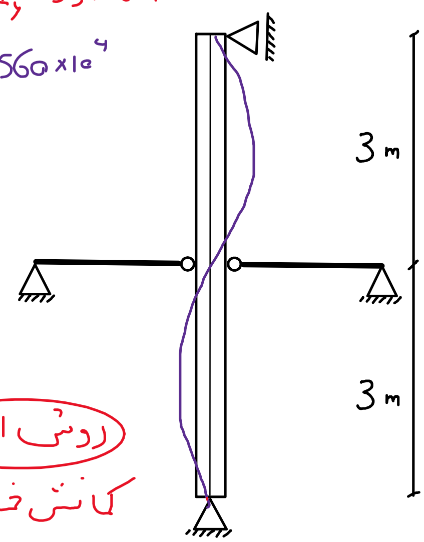
مثال: ستون شکل زیر با مقطع IPB300 از فولاد S355 ساخته شده است. فرض کنید مهاربندی جانبی در وسط ستون، مانع پیچش مقطع می‌شود. مقاومت طراحی ستون را به دست آورید.

IPB 300: $A = 149\times10^2$ $r_x = 130$ $r_y = 75.8$ $I_p = I_x + I_y = 33730\times10^4$

$C_w = 1{,}688{,}000\times10^6$ $J = 186\times10^4$ $I_x = 25170\times10^4$ $I_y = 8560\times10^4$

$b_f = 300$ $t_f = 19$ $h = 208$ $t_w = 11$





$$\begin{cases} \dfrac{b}{t} = \dfrac{150}{19} = 7.9 \le 0.56\sqrt{\dfrac{E}{F_y}} = 13 \checkmark \\ \dfrac{h}{t_w} = \dfrac{208}{11} = 18.9 \le 1.49\sqrt{\dfrac{E}{F_y}} = 35 \checkmark \end{cases}$$

لاغر نیست

روش اول

کمانش خمشی:

$$\begin{cases} \left(\dfrac{L_c}{r}\right)_x = \dfrac{6000}{130} = 46.2 \checkmark \\ \left(\dfrac{L_c}{r}\right)_y = \dfrac{3000}{75.8} = 39.6 \end{cases} \rightarrow F_e = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2} = \frac{\pi^2\times200\times10^3}{46.2^2} = 924.8\ \text{MPa}$$

کمانش پیچشی:

$$G = \frac{E}{2.6} = 76.9\ \text{GPa}$$

$$F_e = \left(\frac{\pi^2}{L^2}EC_w + GJ\right)\frac{1}{I_p} = \left(\frac{\pi^2}{6000^2}\times200\times10^3\times1{,}688{,}000\times10^6 + 76.9\times10^3\times186\times10^4\right)\times\frac{1}{33730\times10^4} = 698.5\ \text{MPa}$$

$F_{e2} = 698.5$ MPa

چون Fe برای کمانش پیچشی کوچکتر است، کمانش پیچشی در این ستون حاکم شده است.

$\frac{F_y}{F_e} = \frac{355}{6985} = 0.51 < 2.25$ inelastic buckling $F_n = \left[0.658^{\frac{F_y}{F_e}}\right] \times F_y = \left[0.658^{0.51}\right] \times 355 = 286.8$ MPa

$\phi P_n = 0.9 F_n A_g = 0.9 \times 286.8 \times 149 \times 10^2 = 3846$ kN $\quad \boxed{\phi P_n = 3846 \text{ kN}}$

روش دوم

$r_z = \sqrt{\frac{C_w}{I_p} + 0.039 \frac{J L^2}{I_p}} = \sqrt{\frac{1,888,000 \times 10^6}{33730 \times 10^4} + 0.039 \times \frac{186 \times 10^4 \times 6000^2}{33730 \times 10^4}} = 112.9$ mm

$r_x = 130$

$r_y = 75.8$

$r_z = 112.9$

$\left(\frac{L_c}{r}\right)_x = \frac{6000}{130} = 46.2$

$\left(\frac{L_c}{r}\right)_y = \frac{3000}{75.8} = 39.6 \rightarrow \left(\frac{L_c}{r}\right) = \max(46.2, 39.6, 53.1) = 53.1 < 4.71\sqrt{E/F_y} = 112$ inelastic buckling

$\left(\frac{L_c}{r}\right)_z = \frac{6000}{112.9} = 53.1$ کانتی بیشتر حاکم می‌شود.

$F_e = \frac{\pi^2 \times 200 \times 10^3}{53.1^2} = 700$ MPa $\rightarrow F_n = \left[0.658^{\frac{355}{700}}\right] \times 355 = 288$ MPa

$\phi P_n = 0.9 F_n A_g = 0.9 \times 288 \times 149 \times 10^2 = 3862$ $\quad \boxed{\phi P_n = 3862 \text{ kN}}$